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Finite Unified Theories and their predictions

Kopaonik 2002

Standard Model

Very successful — highly non-trivial part of a (more) fundamental Theory of Elem. Part.

BUT

Full of free parameters (~20)

Renormalization → free parameters

Traditional way of reducing the number of parameters

SYMMETRY

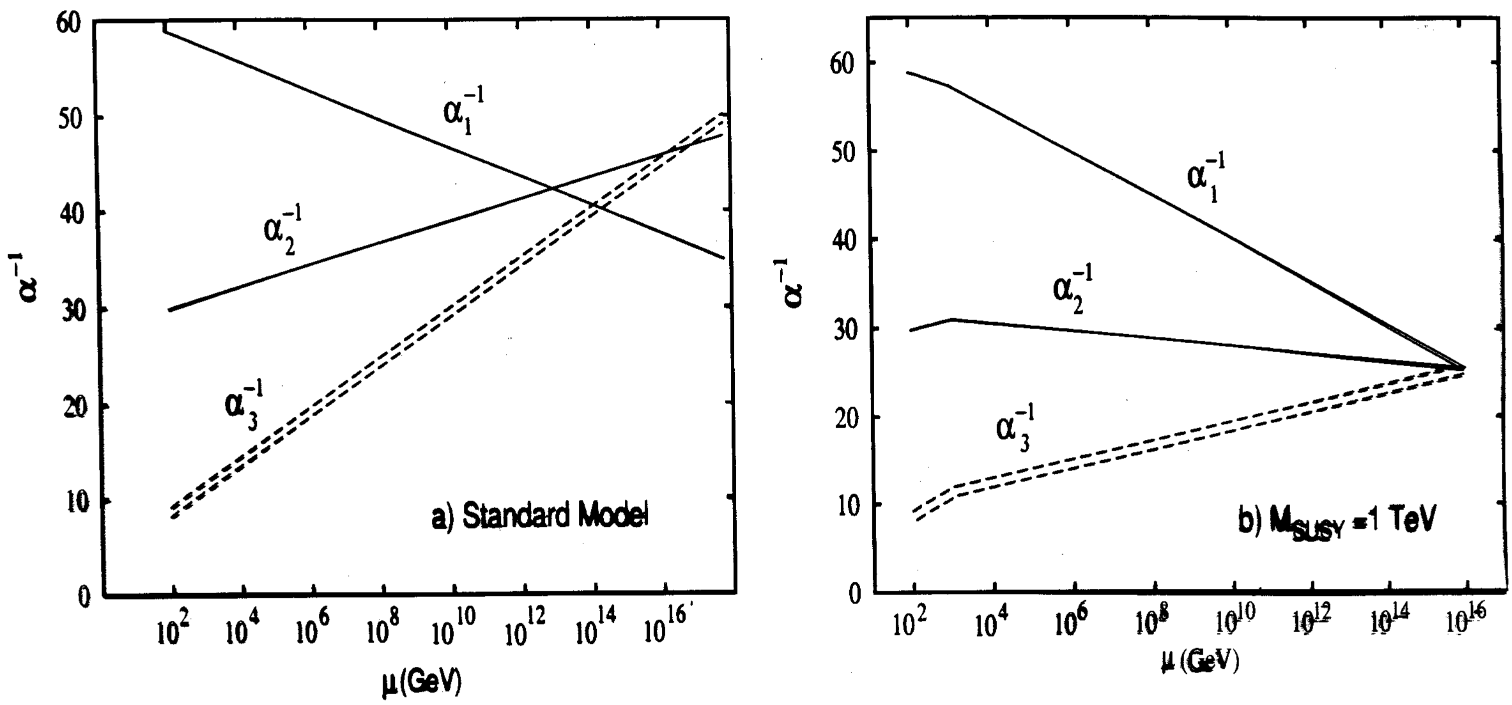
Celebrated example: GUTs

e.g. minimal $SU(5) \rightsquigarrow$ testable $\sin^2\theta_W$

LEP Data $\rightsquigarrow$ $N=1$ $SU(5)$

However more SYMMETRY (e.g. $SO(10)$, $E(6)$, $E(7)$, $E(8)$) does not necessarily lead to more predictions for the SM parameters (due to new degrees of freedom, various ways and channels of breaking etc)

Extreme case: Superstring Th's



60
50
40
30
20
10
0
α^{-1}
α_1^{-1}
α_2^{-1}
α_3^{-1}
a) Standard Model
10^2 10^4 10^6 10^8 10^{10} 10^{12} 10^{14} 10^{16}
μ(GeV)
60
50
40
30
20
10
0
α^{-1}
α_1^{-1}
α_2^{-1}
α_3^{-1}
b) M_{SUSY} =1 TeV
10^2 10^4 10^6 10^8 10^{10} 10^{12} 10^{14} 10^{16}
μ (GeV)

Field Ths on Non-Com Geometries

Heisenberg, Snyder, Connes, Madore, ...

Extension of uncertainty prinziple in spatial coordinates could possibly improve the UV behaviour of field theories.

Recent studies

⤳ UV behaviour of F.T.s on non-com flat space-time is contoled by planar diagrams. In the best case (that are renormalizable) have the same infinities

⤳ IR new phenomena are induced by non-planar diagrams: New problem UV/IR mixing!

GUTs can also relate Yukawa couplings among themselves and might lead to predictions

e.g. in SU_5 $\rightsquigarrow$ successful m_τ/m_b

In SO_{10} all elementary particles of each family (both chiralities + ν_R) are in a common rep 16-plet.

Natural gradual extension:

attempt to relate the couplings of the two sectors

$\rightsquigarrow$ Gauge-Yukawa Unification

Searching for a symmetry is needed one that relates fields with different spins

$\rightsquigarrow$ Supersymmetry   Fayet

BUT $N=2$

Various dimensional reduction schemes (starting with the Coset Space Dim. Red.) suggest that unification of gauge and Higgs fields can be achieved in higher dims.

Addition of fermions in the higher dim thy leads naturally (after CSDR) to Yukawa couplings in 4 dims.

Softly broken supersymmetric ths in 4 dims can be obtained from higher dim susy ths by CSDR over non-symmetric coset spaces.

Various couplings are related classically

GYU – functional relationship derived by some principle.

In • Superstrings
• Composite models

in principle such relations exist.
In practice both have more problems than the S. M.

Attempts to relate gauge and Yukawa couplings:

• Requiring absence of quadratic divergencies (Decker + Pestieau, Veltman)

$$\rightsquigarrow m_e^2 + m_\mu^2 + m_\tau^2 + 3\left(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2\right) = \frac{3}{2} m_W^2 + \frac{3}{4} m_Z^2 + \frac{3}{4} m_H^2$$

• Spontaneous breaking of susy via F-terms

$$\sum_J (-1)^{2J} (2J+1)\, m_J^2 = 0$$

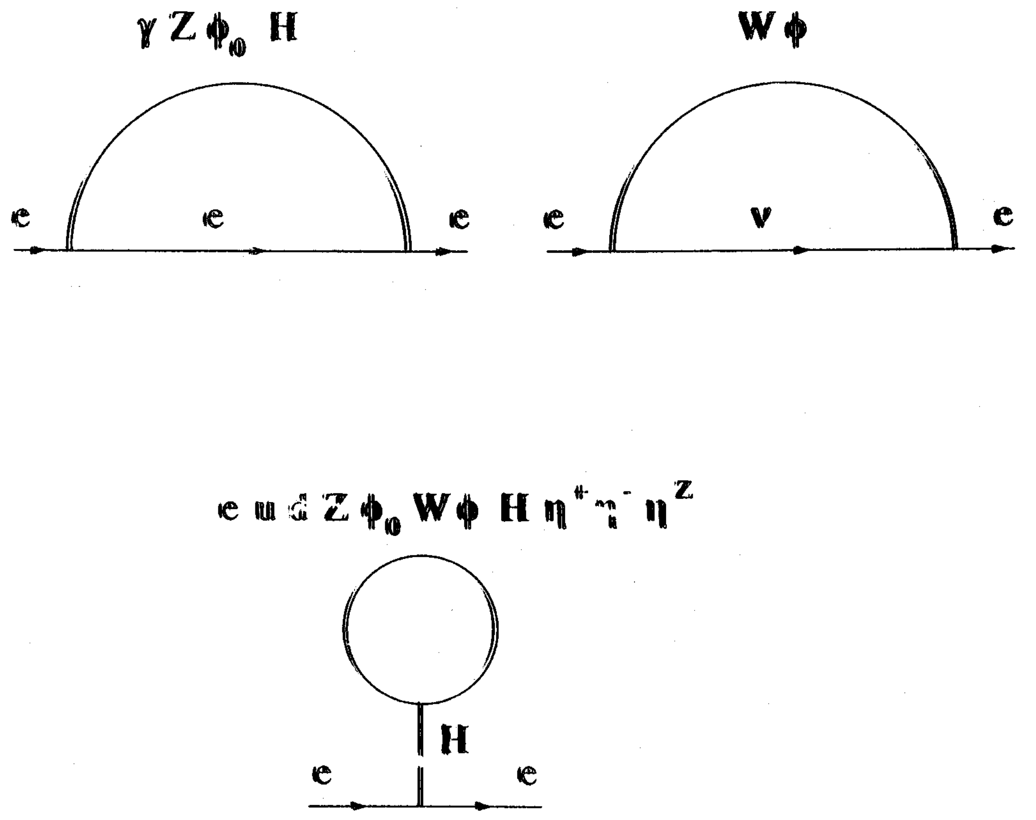


Figure 1

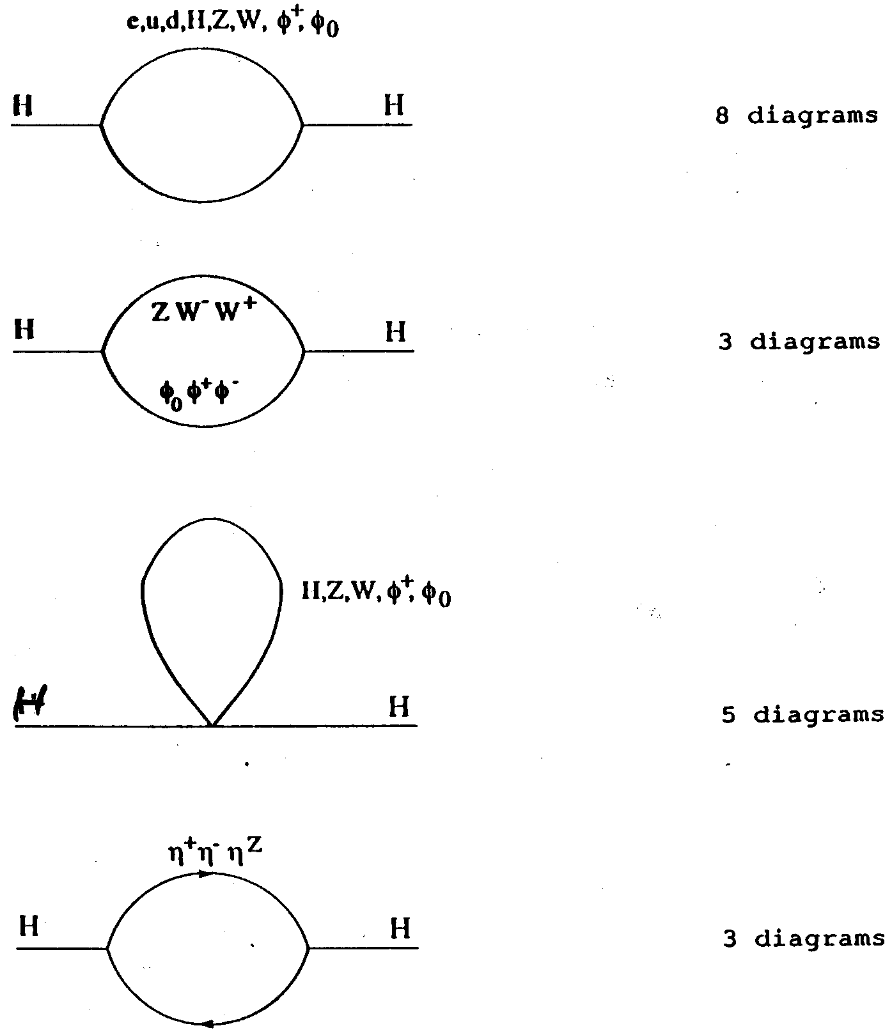


Figure 2

Veltman

Dim. reg. : quadratic div. manifest themselves as pole singularities in $d=2$.

To make them vanish one imposes the condition:

$$\frac{1}{4} f(d)\left[m_e^2 + m_\mu^2 + m_\tau^2 + 3\left(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2\right)\right]$$

$$= \frac{d-1}{2} m_W^2 + \frac{d-1}{4} m_Z^2 + \frac{3}{4} m_H^2$$

where $f(d) = \mathrm{Tr}[1]$

Veltman close $f(d) = 4$ and using susy arguments he put $d=4$

Osland + Wu using point splitting reg

Jack + Jones + Roberts using DRED

found the same relation

Veltman '81

Requiring absence of quadratic divergences found:

$$m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2) = \frac{3}{2} m_W^2 + \frac{3}{4} m_Z^2 + \frac{3}{4} m_H^2$$

For $m_H^2 \ll m_W^2 \rightarrow m_t = 69\,\mathrm{GeV}$

" $m_H^2 = m_W^2 \rightarrow m_t = 77.5\,\mathrm{GeV}$

$\vdots$

$m_H^2 = (316\,\mathrm{GeV})^2 \rightarrow m_t = 174\,\mathrm{GeV}$

Ferrara, Girardello and Palumbo considering the spont. breaking of a susy theory found

$$\sum_J (-1)^{2J} (2J+1) m_J^2 = 0$$

Quiggs

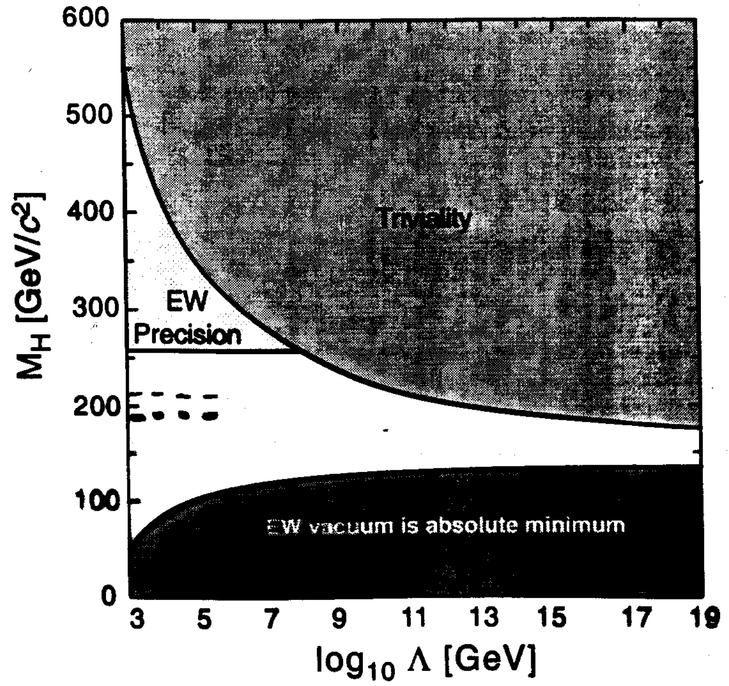


Fig. 4. Bounds on the Higgs-boson mass that follow from requirements that the electroweak theory be consistent up to the energy scale Λ. The upper bound follows from triviality conditions; the lower bound follows from the requirement that $V(v) < V(0)$. Also shown is the range of masses permitted at the 95% confidence level by precision measurements.

where $v \equiv (G_F\sqrt{2})^{-1/2} \approx 246$ GeV is the vacuum expectation value of the Higgs field times $\sqrt{2}$, we find that

$$\Lambda \leq M_H \exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right) . \tag{3.13}$$

For any given Higgs-boson mass, there is a maximum energy scale Λ^* at which the theory ceases to make sense. The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies.

This perturbative analysis breaks down when the Higgs-boson mass approaches 1 TeV/c^2 and the interactions become strong. Lattice analyses [25] indicate that, for the theory to describe physics to an accuracy of a few percent up to a few TeV, the mass of the Higgs boson can be no more than about 710 ± 60 GeV/c^2. Another way of putting this result is that, if the elementary Higgs boson takes on the largest mass allowed by perturbative

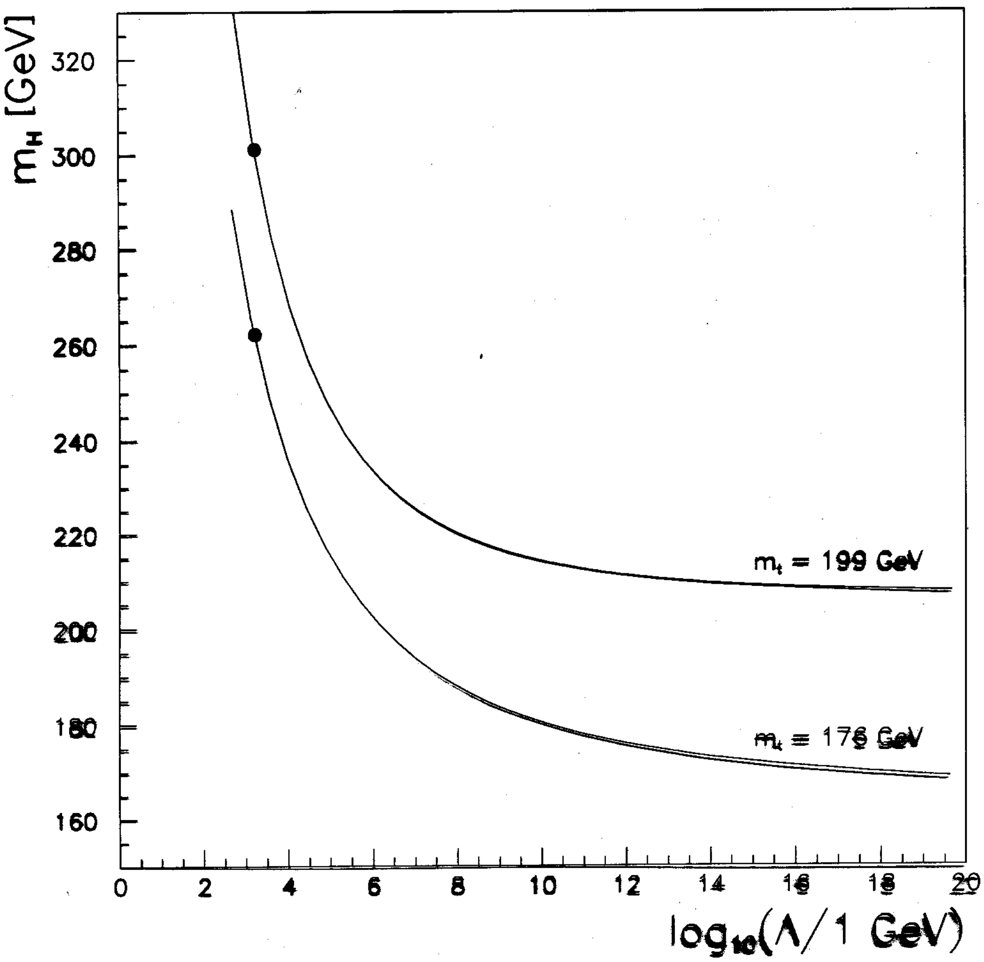


Figure 1: Higgs mass m_H as a function of the scale Λ where cancellation of quadratic divergences is assumed. The bullets denote the intersection points at which the quadratic corrections Δm_H (cf. Eq.(5)) equal the physical mass m_H.

Cancellation of quadratic divergencies

Inami-Nishino-Watanabe

Deshpande-Johnson-Ma

$\downarrow$

SUPERSYMMETRY

unique solution?

SM with two-Higgs doublets

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + (m_3^2 H_1 H_2 + h.c.)$$
$$+ \frac{1}{2}\lambda_1 (H_1^+ H_1)^2 + \frac{1}{2}\lambda_2 (H_2^+ H_2)^2$$
$$+ \lambda_3 (H_1^+ H_1)(H_2^+ H_2) + \lambda_4 (H_1 H_2)(H_1^+ H_2^+)$$
$$+ \left\{\frac{1}{2}\lambda_5 (H_1 H_2)^2 + [\lambda_6 (H_1^+ H_1) + \lambda_7 (H_1^+ H_2^+)](H_1 H_2) + h.c.\right\}$$

Suresymmetry provides tree level relations among couplings

$$\lambda_1 = \lambda_2 = \frac{1}{4}(g^2 + g'^2)$$
$$\lambda_3 = \frac{1}{4}(g^2 - g'^2), \quad \lambda_4 = -\frac{1}{4} g^2$$
$$\lambda_5 = \lambda_6 = \lambda_7 = 0$$

With $v_1 = \langle Re\, H_1^0 \rangle$, $v_2 \equiv \langle Re\, H_2^0 \rangle$

and $v_1^2 + v_2^2 = (246\ GeV)^2$, $\frac{v_2}{v_1} \equiv \tan\beta$

$\Longrightarrow h^0, H^0, H^\pm, A^0$

At tree level

$$M^2_{h^0,H^0} = \frac{1}{2}\left\{M_A^2 + M_Z^2 \mp \left[(M_A^2+M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta\right]^{1/2}\right\}$$

$$M^2_{H^\pm} = M_W^2 + M_A^2$$

$$\Longrightarrow \begin{cases} M_{h^0} < M_Z |\cos 2\beta| \\ M_{H^0} > M_Z \\ M_{H^\pm} > M_W \end{cases}$$

Radiative corrections

$$M^2_{h^0} \simeq M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{16\pi^2 M_W^2} \log \frac{\tilde{m}^2_{t_1} \tilde{m}^2_{t_2}}{m_t^4}$$

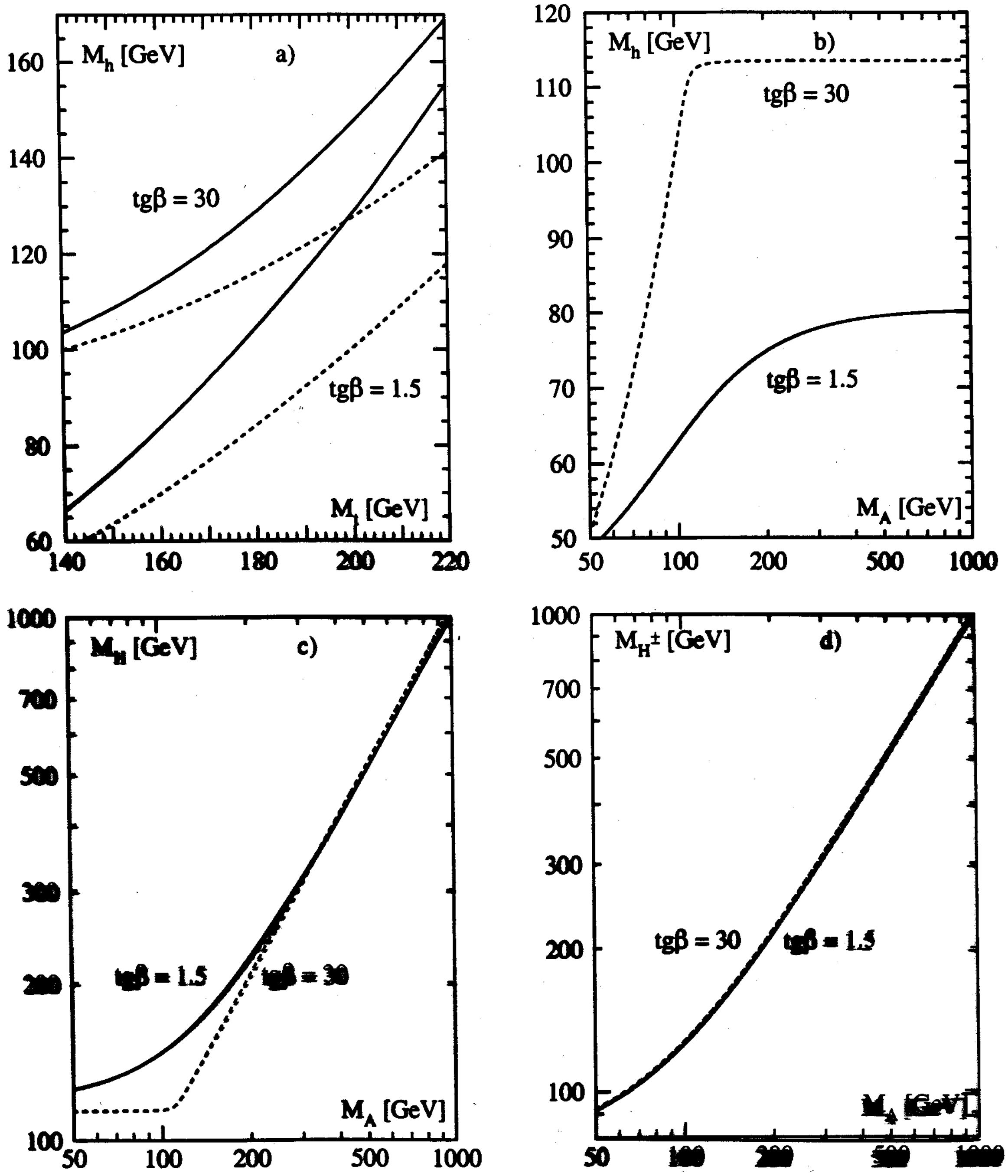


Figure 18: *(a) The upper limit on the light scalar Higgs pole mass in the MSSM as a function of the top quark mass for two values of $tg\beta = 1.5, 30$; the common squark mass has been chosen as $M_S = 1$ TeV. The full lines correspond to the case of maximal mixing $[A_t = \sqrt{6}M_S,\ A_b = \mu = 0]$ and the dashed lines to vanishing mixing. The pole masses of the other Higgs bosons, $H, A, H^\pm$, are shown as a function of the pseudoscalar mass in (b–d) for two values of $tg\beta = 1.5, 30$, vanishing mixing and $M_t = 175$ GeV.*

- Pendleton - Ross infrared fixed point

$$\frac{d}{dt}\left(Y_t / \alpha_3\right) = 0 \rightsquigarrow m_t \sim 100 \text{ GeV}$$

(Divergent! in 2-loops, Zimmermann)

- Infrared quasi-fixed point (Hill)

$$\frac{d}{dt}\left(Y_t\right) = 0 \rightsquigarrow m_t \sim 280 \text{ GeV}$$

$$Y_t \equiv \frac{h_t^2}{4\pi}$$

- Susy Pendleton - Ross

$$\rightsquigarrow m_t \simeq 140 \text{ GeV} \cdot \sin\beta$$

$$\tan\beta = \frac{v_U}{v_D}$$

- Susy Quasi-fixed point (Barger et. al. ; Carena et. al.)

$$\rightsquigarrow m_t \simeq 200 \text{ GeV} \cdot \sin\beta$$

(If $\tan\beta > 2 \rightsquigarrow m_{t\,I.R.} \gtrsim 188$ GeV Kubo et. al.)

Standard Model

- Pendleton-Ross infrared fixed point:

For strong α_3, i.e. $\alpha_1 = \alpha_2 = 0$

$$\frac{d\alpha_3}{dt} = -7\alpha_3^2$$

$$\frac{dY_t}{dt} = -\frac{Y_t}{4\pi}\left(8\alpha_3 - \frac{9}{2}Y_t\right) \; ; \; Y_t = \frac{h_t^2}{4\pi}$$

$$P-R : \frac{d}{dt}\left(Y_t/\alpha_3\right) = 0 \rightsquigarrow Y_t = \frac{2}{9}\alpha_3$$

$$\rightsquigarrow m_t^{P-R} = \sqrt{\frac{8\pi}{9}\alpha_3} \cdot v \sim 100 \text{ GeV}$$

In the reduction scheme same result is obtained from the requirement that the system is described by a single coupling thy with a renormalized power series expansion in α_3.

•• Infrared Quasi-fixed point:

Vanishing β-function for Y_t

$$\rightsquigarrow \frac{9}{2} Y_t^{Q-f} = 8\,\alpha_3$$

$$\rightsquigarrow m_t^{Q-f} = \sqrt{8}\, m_t^{P-R} \sim 280\ \text{GeV}$$

* Quasi-fixed point would also become an exact fixed point if $\beta_3 = 0$.

Susy S. M.

- Pendleton – Ross:

$$\frac{d\alpha_3}{dt} = -3\alpha_3^2$$

$$\frac{dY_t}{dt} = Y_t\left(\frac{16}{3}\frac{\alpha_3}{4\pi} - 6Y_t\right)$$

$$\frac{d}{dt}\left(Y_t/\alpha_3\right) = 0 \rightsquigarrow Y_t^{susy\,P-R} = \frac{7}{18}\alpha_3$$

$$\rightsquigarrow m_t^{susy\,P-R} = \sqrt{\frac{7}{18}4\pi\,\alpha_3}\cdot v\cdot \sin\beta$$

$$\sim 140\ \mathrm{GeV}\cdot\sin\beta$$

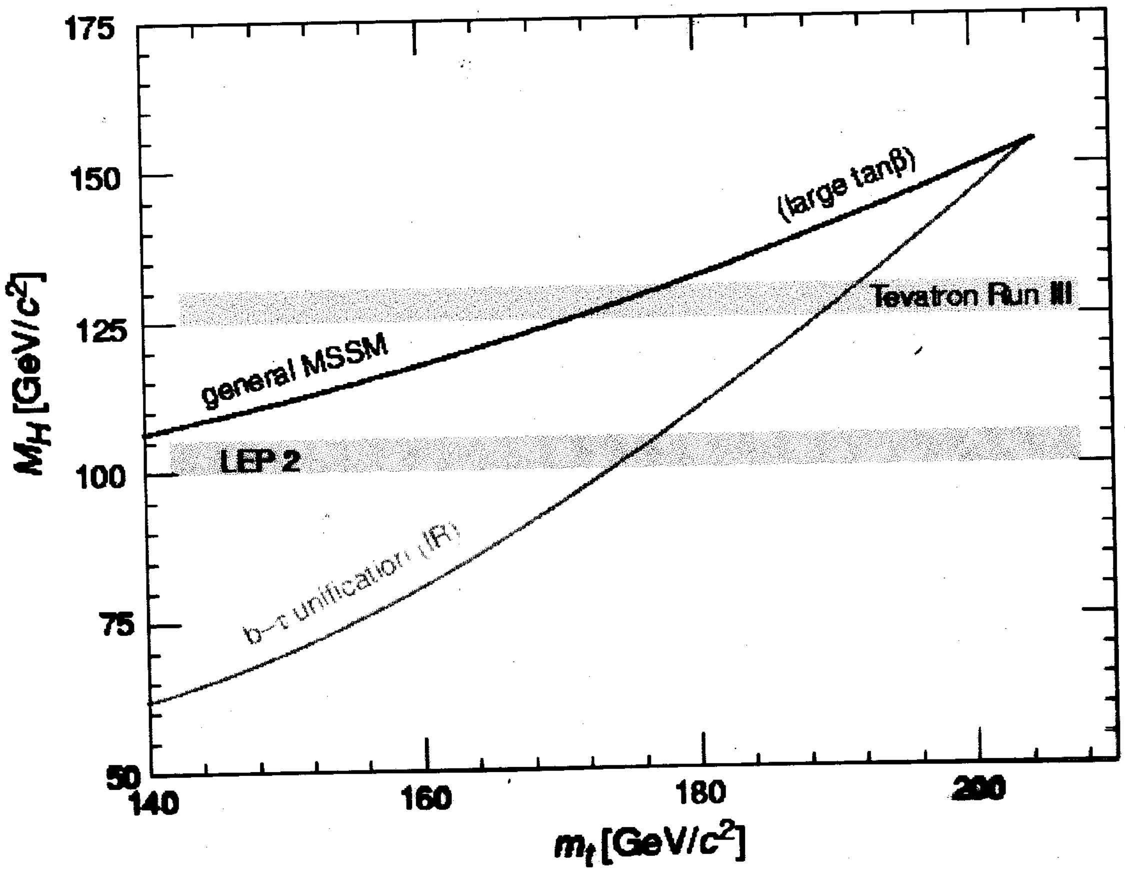
$$\tan\beta = \frac{v_u}{v_D}$$

- • Quasi-fixed point:

$$Y_t^{susy\,Q-f} = \frac{16}{18}\frac{\alpha_3}{4\pi}$$

$$\rightsquigarrow m_t^{susy\,Q-f} \sim 200\ \mathrm{GeV}\cdot\sin\beta$$

Quasi-fixed point is reached if h_t becomes strong at scales $\mu = 10^{14} - 10^{19}$ GeV



Carena
Espinosa
Quiros
Wagner

Fig Upper bounds on the mass of the lightest Higgs boson, as a function of the top-quark mass, in two variants of the minimal supersymmetric standard model. The upper curve refers to a general MSSM, in the large-tan β limit; the lower curve corresponds to an infrared-fixed-point scenario with b-τ unification, from Ref. [illegible].

We attempt to reduce further the parameters of GUTs searching for renormalization group invariant relations among GUT's couplings holding beyond the unification scale.

- Finite SU(5)
  Kapetanakis, Mondragon, Z — '92, '93
- Minimal Susy SU(5)
  Kubo, Mondragon, Z — '94
- Susy Pati-Salam $SU(4)\times SU(2)_L \times SU(2)_R$
  Kubo, Mondragon, Tracas, Z — '94
- Susy SO(10)
  Kubo, Mondragon, Shoda, Z — '95
- Testing reduction of parameters by m_t
  Kubo, Mondragon, Olechowski, Z — '95, '96
- Reduction of Susy breaking parameters
  Kubo, Mondragon, Z + Kobayashi — '96, '97, '98, '99, '00, '01, 02
- Dualities of finite th's
  Karsh, Luest, Z, Kobayashi, Kubo — '97, '98

In a GUT with

g - gauge coupling

g_i - other couplings (Yukawas, self-couplings)

Any R.G.I. relation among the couplings can be expressed as

$$\Phi(g, g_1, \dots) = \text{const}$$

$$\rightsquigarrow \frac{d}{dt}\Phi = 0 \quad , \quad t = \ln\mu$$

$$\frac{d\Phi}{dt} = \frac{\partial\Phi}{\partial g}\underbrace{\frac{dg}{dt}}_{\equiv\, b_g} + \sum_i \frac{\partial\Phi}{\partial g_i}\underbrace{\frac{dg_i}{dt}}_{\equiv\, b_i} = 0$$

This is math. equivalent to

$$\frac{dg}{b_g} = \frac{dg_1}{b_1} = \frac{dg_2}{b_2} = \dots$$ ~~characteristic~~ ~~system~~

$$\Longrightarrow \quad b_g \frac{dg_i}{dg} = b_i \quad \Bigg\| \quad \text{Reduction eqs}$$

The strongest requirement is to demand the formal power series solutions to the REs

$$g_i = \sum_{n=0} \rho_i^{(n+1)} g^{2n+1}$$

Remarkably; uniqueness of these solutions can be decided already at 1-loop!

Assume

$$\beta_i = \frac{1}{16\pi^2}\left[\sum_{j,k,l\neq g} \beta_i^{(1)} g_j\, g_k\, g_l + \sum_{j\neq g} \beta_i^{(1)j} g_j\, g^2\right] + \cdots$$

$$\beta_g = \frac{1}{16\pi^2} \beta_g^{(1)} g^3 + \cdots$$

Assume $\rho_i^{(n)}$, $n \leq r$ have been uniquely determined

To obtain $\rho_i^{(r+1)}$, insert g_i in REs and collect terms of $O(g^{2r+3})$

$$\rightsquigarrow \sum_{\ell \neq g} M(r)_i^{\ell}\, \rho_\ell^{(r+1)} = \text{lower order terms known by assumption}$$

$$M(r)_i^{\ell} = 3 \sum_{j,k \neq g} B_i^{(1)} \rho_j^{(1)} \rho_k^{(1)} + B_i^{(1)\ell} - (2r+1) B_g^{(1)} \delta_i^{\ell}$$

$$0 = \sum_{j,k,\ell \neq g} B_i^{(1)jk\ell} \rho_j^{(1)} \rho_k^{(1)} \rho_\ell^{(1)} + \sum_{\ell \neq g} B_i^{(1)\ell} \rho_\ell^{(1)} - B_g^{(1)} \rho_i^{(1)}$$

$\rightsquigarrow$ $\rho_i^{(n)}$ for all $n > 1$ for a given set of $\rho_i^{(1)}$ can be uniquely determined if

$$\det M(n)_i^{\ell} \neq 0 \text{ for all } n \geq 0$$

Kubo et.al.

Consider an $SU(N)$ (non-susy) theory with

$\phi^i(N)$, $\hat{\phi}_i(\bar{N})$ - complex scalars:

$\psi^i(N)$, $\hat{\psi}_i(\bar{N})$ - Weyl spinors

$\lambda^a \; (a = 1, \dots, N^2-1)$ - "

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + i\sqrt{2}\,\Big[g_Y \bar{\psi} \lambda^a T^a \phi - \hat{g}_Y \bar{\hat{\psi}} \lambda^a T^a \hat{\phi} + h.c. - V(\phi, \hat{\phi}),$$

$$V(\phi, \hat{\phi}) = \frac{1}{4} \lambda_1 (\phi^i \phi_i^*)^2 + \frac{1}{4} \lambda_2 (\hat{\phi}_i \hat{\phi}^{*i})^2 + \lambda_3 (\phi^i \phi_i^*)(\hat{\phi}_j \hat{\phi}^{*j}) + \lambda_4 (\phi^i \phi_j^*)(\hat{\phi}_i \hat{\phi}^{*j})$$

Searching for power series solution of the R.E.s we find

$$g_Y = \hat{g}_Y = g; \; \lambda_1 = \lambda_2 = \frac{N-1}{N} g^2; \; \lambda_3 = \frac{1}{2N} g^2; \; \lambda_4 = -\frac{1}{2} g^2$$

i.e. SUSY

$N=1$ gauge theories

Consider a chiral, anomaly free $N=1$ globally supersymmetric gauge th. based on a group G with gauge coupling g.

Superpotential

$$W = \frac{1}{2} m_{ij} \phi^i \phi^j + \frac{1}{6} C_{ijk} \phi^i \phi^j \phi^k$$

m_{ij}, C_{ijk} – gauge invariant tensors

ϕ^i – matter fields transforming as an ir. rep. R_i of G.

Renormalization constants associated with W

$$\phi^{0i} = (Z^i_j)^{1/2} \phi^j \,, \quad m^0_{ij} = Z^{i'j'}_{ij} m_{i'j'} \,, \quad C^0_{ijk} = Z^{i'j'k'}_{ijk} C_{i'j'k'}$$

$N = 1$ non-renormalization thm ensures absence of mass and cubic-int-term infinities

$$Z^{ijk}_{i'j'k'} Z^{1/2\, i'}_{i''} Z^{1/2\, j'}_{j''} Z^{1/2\, k'}_{k''} = \delta^i_{(i''} \delta^j_{j''} \delta^k_{k'')}$$

$$Z^{ij}_{i'j'} Z^{1/2\, i'}_{i''} Z^{1/2\, j'}_{j''} = \delta^i_{(i''} \delta^j_{j'')}$$

(In the background field method)

$$Z_g Z_V^{1/2} = 1$$

$\rightarrow$ Only surviving infinities are $Z^i_j, (Z_V)$

i.e. one infinity for each field.

The 1-loop β-function of the gauge coupling is

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$\ell(R_i)$ – Dynkin index of R_i

$C_2(G)$ – quadratic Casimir of the adjoint rep.

β-functions of C_{ijk}, by virtue of non-renormalization thm, are related with the anomalous dim. matrix γ_{ij} of ϕ^i

$$\beta_{ijk}^{(1)} = \frac{dC_{ijk}}{dt} = C_{ijl}\gamma^l_k + C_{ikl}\gamma^l_j + C_{jkl}\gamma^l_i$$

$$\gamma_i^{(1)j} = Z^{-\frac{1}{2}}{}^k_i \frac{d}{dt} Z^{\frac{1}{2}}{}^j_k$$

$$= \frac{1}{32\pi^2}\left[C^{jkl}C_{ikl} - 2g^2 C_2(R_i)\delta^j_i\right]$$

$C_2(R_i)$ – quadratic Casimir of R_i

$C^{ijk} = C^*_{ijk}$

$$\ell(R_i)\,\delta_{ab} = \mathrm{Tr}\,(T_a T_b)$$

$$C_2(R_i)\,\delta_{ij} = \sum_a (T^a T^a)_{ij}$$

where $T_a\,[a = 1, \dots, \dim(adj)]$ is the a-th generator in the R_i rep.

$$C_2(R_i) = \frac{\dim(adj)}{\dim(R_i)}\,\ell(R_i) \qquad //$$

e.g. in $SU(N)$

N has $\ell = 1$ $\rightsquigarrow$ $C_2(N) = \frac{N^2-1}{N}$

N^2-1 " $\ell = 2N$ $\rightsquigarrow$ $C_2(N^2-1) = 2N$

$$b_g^{(2)} = \frac{1}{(16\pi^2)^2}\, 2\, g^5 \left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$$- \frac{1}{(16\pi^2)^2}\, \frac{g^3}{r}\, C_2(R_i)\left[C^{jkl} C_{ikl} - 2g^2 C_2(R_i)\delta^j_i\right]$$

Parkes, West, Jones

$r : tr\,\delta^{ab}$ Mezincescu, Yau

Machacek, Vaughn

$$\gamma^{(2)i}_{\ \ j} = \frac{1}{(16\pi^2)^2}\ 2\, g^4\, C_2(R_i)\left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$$- \frac{1}{(16\pi^2)^2}\, \frac{1}{2}\left[C^{ikl} C_{jkm} + 2g^2 (R^a)^i_{\ m} (R^a)^l_{\ j}\right]$$

$$\cdot \left[C^{mpq} C_{lpq} - 2\,\delta^m_l\, g^2 C_2(R_i)\right]$$

$$b_g^{NSVZ} = \frac{g^3}{16\pi^2}\left[\frac{\sum_i \ell(R_i)(1 - 2\gamma_i) - 3\,C_2(G)}{1 - g^2 C_2(G)/8\pi^2}\right]$$

Novikov – Shifman – Vainstein – Zakharov

Wilsonian Renorm. Group (WRG)

Any field theory is defined with cutoff M and bare couplings λ^i_0. If we wish to change $M \to M'$ i.e. integrate out modes between M and M' and keep low energy physics fixed we need to change

$$\lambda^0_i \to \lambda^0_i$$

Necessary changing of λ^0_i is encoded in a WRGE

$$M \frac{d\lambda^0_i}{dM} = \beta_i(\lambda^0_i)$$

A $N=1$ pure Yang-Mills with vector multiplet $V_h = V_h^a T^a$ can be defined at M as

- $$\mathcal{L}_h^M(V_h) = \frac{1}{16}\int d^2\theta \, \frac{1}{g_h^2} W^\alpha(V_h) W^\alpha(V_h) + h.c.$$

where $\frac{1}{g_h^2} = \frac{1}{g^2} + i\frac{\theta}{8\pi^2}$

manifestly holomorphic in g_h

- • $$\mathcal{L}_c^M(V_c) = \frac{1}{16}\int d^2\theta \left(\frac{1}{g_c} + i\frac{\theta}{8\pi^2}\right) W^\alpha(g_c V_c) W^\alpha(g_c V_c) + h.c.$$

with canonical normalization

Analyticity arguments

$\rightsquigarrow \beta\left(\frac{8\pi^2}{g_h^2}\right) = b_0$

Holomorphic $1/g_h^2$ runs exactly at 1-loop even including non-perturbative effects.

To determine the Wilsonian b-function for the canonical g_c it is not enough to change variables of the holomorphic $\mathcal{L}_h^M(V_h)$ $V_h = g_c V_c$ to obtain the $\mathcal{L}_c^M(V_c)$ with $g_c = g_h$. There is an anomalous Jacobian in passing from V_h to V_c.

$$\Rightarrow \frac{1}{g_c^2} = \mathrm{Re}\left(\frac{1}{g_h^2}\right) - \frac{2C_2(G)}{8\pi^2} \ln g_c$$ Arkani-Hamed Murayama

$$\rightsquigarrow M\frac{d}{dM} g_c = b(g_c) = -\frac{3C_2(G)}{16\pi^2} \frac{g_c^3}{1 - \frac{C_2(G) g_c^2}{8\pi^2}}$$

In presence of matter fields generalizes to the full Novikov, Shifman, Vainshtein, Zakharov all-loop b-function.

Finite Unification

Old days...

... divergences are "hidden under the carpet" (Dirac, Lects on Q.F.T., '64)

Recent years ...

... divergences reflect existence of a higher scale where new degrees of freedom are excited. Not just artifacts of pert. th.

However the presence of quadratic divergences means that physics at one scale are very sensitive to unknown physics at higher scales.

⇝ SUSY ths which are free of quadratic divergences in spite of any experimental evidence...

⇝ Natural to expect that beyond unification scale the theory should be completely finite.

- $N=4$ ⇝ finite to all orders in pert.
- $N=2$ ⇝ only 1-loop contributions to β-function. Possible to arrange the spectrum so that theory is finite.

Multiplicities for massless irreducible reps with maximal helicity 1

N / Spin	1	1	2	2	4
1	–	1	–	1	1
1/2	1	1	2	2	4
0	2	–	4	2	6

$$N=2 \ : \ \beta(g) = \frac{2g^3}{(4\pi)^2}\left(\sum_i T(R_i) - C_2(G)\right)$$

e.g. $SU(N)$ with $2N$ fundamental reps $\rightarrow \beta(g) = 0$

$SU(5)$: $p(5+\bar{5})$; $q(10+\overline{10})$; $r(15+\overline{15})$

with $p+3q+7r = 10$

$SO(10)$: $p(10+\overline{10})$; $q(16+\overline{16})$

with $p+2q = 8$

E_6 : $4(27+\overline{27})$

Finite Unified Theories

$N=1$

• 1 - loop finiteness conditions

$b_g^{(1)} = 0$

$\gamma^{(1)i}_{\ \ j} = 0$ - anomalous dimensions of all chiral superfields

•• Exists complete classification of all chiral $N=1$ models with $b_g^{(1)} = 0$ Hamidi - Patera - Schwarz

Jiang - Zhou

••• 1 - loop finiteness $\longrightarrow$ 2 - loop finiteness Parkes-West Jones Mezincescu

.... Exist simple criteria that guarantee all loop finiteness (vanishing of all-loop beta functions)

Lucchesi-Piguet
Sibold

Ermushev
Kazakov
Tarasov

Leigh-Strassler

- All-loop finite SU(5) $\Longrightarrow$ top quark mass ✓

Kapetanakis
Mondragon
Z
'92

~~Susy~~ sector

Jones
Mezincescu
Yao

- 1-loop finiteness conds (require in particular universal soft ~~susy~~ scalar masses

$$(m^2)^i_j = \frac{1}{3} M M^* \delta^i_j \Big)$$

·· 1-loop finiteness

→ 2-loop finiteness — Jack, Jones

Reduction of couplings

- Extension of method in SSB sector + application in min susy SU(5) — Kubo, Mondragon, Z

·· 1-loop sum rule for soft scalar masses in non-finite susy ths. — Kawamura, Kobayashi, Kubo

··· 2-loop sum rule for soft scalar masses in finite ths. — Kobayashi, Kubo, Mondragon, Z

* All-loop RGI relations in finite and non-finite ths — Yamada, Hisano, Shifman, Kazakov, Jack, Jones, Pickering

** All-loop sum rule for soft scalar masses in finite and non-finite ths

Kobayashi
Kubo
Z

•• SU(5) FUTs

Kobayashi
Kubo
Mondragon
Z

· Prediction of s-spectrum in terms of few parameters starting from few hundreds GeV

·· The LSP is neutralino ✓ (see e.g. Kazakov et. al. Yoshioka)

··· Radiative E-W breaking ✓ (see e.g. Brignole Ibanez, Munoz)

···· No funny colour, charge ✓ (see e.g. Cases et. al)

* Prediction of Higgs masses

Lightest ~ 120 – 130 GeV

Similar results also for min susy SU(5)

Consider a chiral, anomaly free, $N=1$ gauge theory with group G. The superpotential is

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

$\left.\begin{array}{l} Y^{ijk} \\ \mu^{ij} \end{array}\right\}$ gauge invariant

Yukawa couplings

Φ_i – matter superfields in irreducible reps of G

Necessary and sufficient conditions for $N=1$ 1-loop finiteness

- Vanishing of $b_g^{(1)}$ implies

$$\sum_i \ell(R_i) = 3 C_2(G)$$

$\ell(R_i)$ – Dynkin index of R_i
$C_2(G)$ – Quadratic Casimir of G (adjoint)

$\Rightarrow$ Selection of the field content (representations) of the theory

CHIRAL TWO-LOOP-FINITE SUPERSYMMETRIC THEORIES ☆

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Received 2 April 1984

Any globally supersymmetric theory in four dimensions that is one-loop finite is automatically (at least) two-loop finite. We classify all such theories that are chiral and have a simple gauge group.

One of the less satisfying aspects of GUTs ("grand-unified theories") and super GUTs is that they contain many arbitrary parameters. One principle that could serve to limit the number of parameters is the requirement of finiteness. This possibility has been raised with the discovery that there are large classes of supersymmetric gauge theories that are free from ultraviolet divergences at all orders of perturbation theory [*1]. The theories for which this has been established are all $N = 4$ and some [2] $N = 2$ super Yang–Mills theories. Finiteness allows some parameters to be introduced via mass and soft supersymmetry-breaking terms, but it relates all the dimensionless couplings. Unfortunately, all $N = 2$ or $N = 4$ theories are nonchiral ("vector-like") and do not appear suited to the construction of a realistic model. Recently, the possibility has been raised that certain $N = 1$ theories could also be finite [3,4]. A theory containing Yang–Mills and chiral superfields is ultraviolet finite at loop provided that certain conditions (described below) restricting the representations and couplings of the chiral superfields are satisfied. It has been proved by direct calculation [3] and by considerations involving the chiral anomaly [4] that these conditions ensure two-loop finiteness as well, without any additional restrictions. It is an open question whether any of the $N = 1$ theories of this class are finite beyond two loops. The purpose of this letter is to list all chiral solutions of the one-loop conditions that are based on a simple gauge group.

Consider a globally supersymmetric $N = 1$ theory in four dimensions with a simple Yang–Mills group G. In addition to the gauge superfield, it can contain chiral superfields in an arbitrary representation R of G with irreducible components R_i:

$$\mathrm{R} = \bigoplus_i \mathrm{R}_i . \tag{1}$$

Our task is to find the possible choices of R and associated couplings that ensure one-loop finiteness. We only consider chiral theories ($\mathrm{R} \neq \bar{\mathrm{R}}$). This restricts G to those groups that have complex representations, namely SU(n) with $n \geqslant 3$, SO($4k + 2$) with $k \geqslant 2$, and E_6. Cancellation of the gauge-current anomaly

$$A(\mathrm{R}) = \sum_i A(\mathrm{R}_i) = 0 \tag{2}$$

is also imposed, since it is a necessary requirement for a consistent quantum theory. The anomaly condition is nontrivial only for SU(n).

There are two additional conditions required by one-loop finiteness [3,4]. The first is the one-loop finiteness of the gauge-field self energy. The condition is

$$l(\mathrm{R}) = \sum_i l(\mathrm{R}_i) = 3C_2(\mathrm{G}) , \tag{3}$$

where $l(\mathrm{R}_i)$ is the "index" of R_i [5] and $C_2(\mathrm{G})$ is

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[*1] For a review see ref. [1].

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the eigenvalue of the second-order Casimir operator (which coincides with the index of the adjoint representation). Since indices are always positive (except for singlets, which are excluded), eq. (3) already limits R to a finite number of possibilities for a given group G.

The second condition is the one-loop finiteness of the chiral superfield self-energy. In terms of coefficients d describing the cubic self-coupling of the chiral superfields in the superpotential, the condition is

$$\sum_{\substack{b,c\\ j,k}} d_{abc}^{ijk}\,\bar{d}_{a'bc}^{i'jk} = 2g^2\delta_{aa'}\delta_{ii'}C_2(\mathrm{R}_i)\,. \tag{4}$$

The subscripts a, b, c label components of the representations $\mathrm{R}_i, R_j, \mathrm{R}_k$.

The only irreducible representations that can occur in R are ones whose indices do not exceed $3C_2(\mathrm{G})$. Singlets (with nonzero couplings) are excluded by (4). All relevant representations of the groups with complex representations are listed in table 1. It also gives the indices and anomalies, normalized to be unity for the fundamental representation. Complex-conjugate representations, which have the same index and opposite anomaly, are not shown.

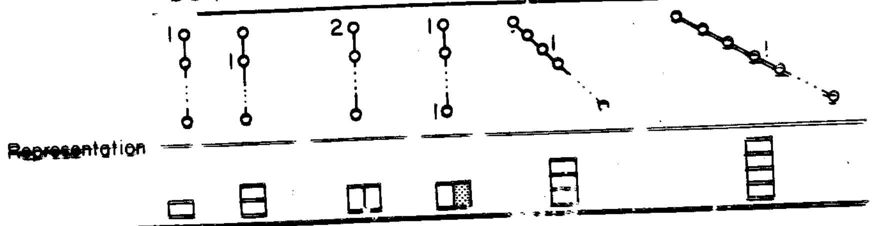
In seeking solutions to eqs. (2)–(4) it is convenient to consider first the trace of (4) given by summing over $a = a'$ and the m_α values of $i = i'$ for which $\mathrm{R}_i = \mathrm{R}_\alpha$. This results in conditions of the form

$$\sum_{\beta\gamma} |C_{\alpha\beta\gamma}|^2 = m_\alpha I(\mathrm{R}_\alpha)\,. \tag{5}$$

Eq. (5) is weaker than (4), but it is useful for quickly eliminating many candidates from the list of admissible R's. Detailed examination of (4) then eliminates

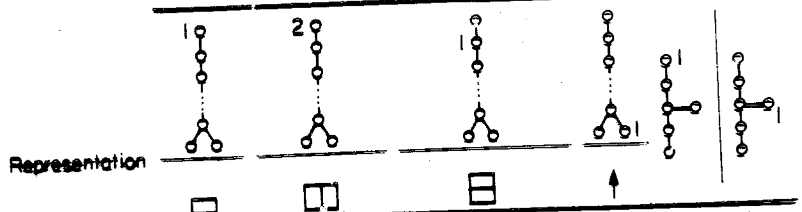
Table 1
Properties of relevant irreducible representations

SU(n)



Dimension	n	n(n−1)/2	n(n+1)/2	n^2-1	n(n−1)(n−2)/6	n(n−1)(n−2)(n−3)/24
Index	1	n−2	n+2	2n	(n−2)(n−3)/2	(n−2)(n−3)(n−4)/6
Anomaly	1	n−4	n+4	0	(n−3)(n−6)/2	(n−3)(n−4)(n−8)/6

O(4k+2) E(6)



Dimension	4k+2	2(k+1)(4k+1)	(4k+2)(4k+1)/2	2^{2k}	27	78
Index	1	4k+4	4k	2^{2k-3}	1	4

Table 2
Multiplicities m_α of the irreducible components R_α of R for all the solutions.

Irrep	27	$\overline{27}$	Comments
E(6)	n	$12-n$	$7 \leqslant n \leqslant 12$

Irrep	10	54	45	16	$\overline{16}$	Comments
SO(10)	8	0	0	n	$8-n$	$5 \leqslant n \leqslant 8$
	2	1	1	1	0	
	$12-2m-2n$	1	0	n	m	$n+m \leqslant 4$, $n > m$

Irrep	$\square$	$\overline{\square}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$	$\square\square$	$\overline{\square\square}$	Adj
SU(n)	$2n-4$	$2n+4$	0	1	1	0	0
$n \geqslant 7$	$n-4$	$n+4$	0	1	1	0	1

Irrep	8	$\overline{8}$	28	70	63
SU(8)	1	5	1	1	1

Irrep	3	$\overline{3}$	6	8
SU(3)	3	10	1	0
	0	7	1	1

Irrep	4	$\overline{4}$	15	6	10
SU(4)	0	8	1	1	1
	4	12	0	1	1

Irrep	5	$\overline{5}$	10	$\overline{10}$	15	$\overline{15}$	24
SU(5)	3	14	2	0	1	0	0
	6	14	0	1	1	0	0
	5	7	4	2	0	0	0
	5	10	5	0	0	0	0
I →	6	9	4	1	0	0	0
	7	8	3	2	0	0	0
	8	10	3	1	0	0	0
	1	2	1	0	1	1	1
	1	9	0	1	1	0	1
	2	3	3	2	0	0	1
	3	5	3	1	0	0	1
II →	4	7	3	0	0	0	1
	5	6	2	1	0	0	1
	6	8	2	0	0	0	1
	8	9	1	0	0	0	1
	3	4	1	0	0	0	2

Irrep	6	$\overline{6}$	15	$\overline{15}$	21	$\overline{21}$	20	35
SU(6)	0	16	3	0	1	0	0	0
Sibold et al. →	8	16	0	1	1	0	0	0
	0	4	5	3	0	0	0	0
	3	5	4	3	0	0	0	0
	0	12	6	0	0	0	0	0
	2	10	5	1	0	0	0	0
	4	8	4	2	0	0	0	0
	8	12	3	1	0	0	0	0
	0	2	1	0	0	0	3	1
	0	4	2	0	0	0	2	1
	3	5	1	0	0	0	2	1
	0	6	3	0	0	0	1	1
	2	4	2	1	0	0	1	1
	3	7	2	0	0	0	1	1
	6	8	1	0	0	0	1	1
	1	3	1	0	1	1	0	1
	2	10	0	1	1	0	0	1
	1	3	3	2	0	0	0	1
	0	8	4	0	0	0	0	1
	2	6	3	1	0	0	0	1
	3	9	3	0	0	0	0	1
	5	7	2	1	0	0	0	1
	6	10	2	0	0	0	0	1
	9	11	1	0	0	0	0	1
	0	4	2	0	0	0	0	2
	3	5	1	0	0	0	0	2
	0	2	1	0	0	0	1	2

some [1 for SU(5) and 11 for SU(6)] of the class allowed by (2), (3), and (5). The complete list of complex representations satisfying (2), (3) and (4) is given in table 2. The conjugate representations, which are also solutions, are not tabulated. In most cases the couplings are uniquely determined (up to a change of basis), but in a few cases there are free parameters or discrete alternatives. Note that there are no solutions for SO($4k+2$) with $k > 2$.

Scanning the tables for potentially realistic schemes,

•• Vanishing of $\gamma^{(1)i}_{\;j}$ implies

$$\underset{\substack{\uparrow \\ \text{Yukawa}}}{Y^{ikl}}\, Y_{jkl} = 2\,\delta^i_j\, \underset{\substack{\uparrow \\ \text{gauge}}}{g^2}\, C_2(R_i)$$

$C_2(R_i)$ – quadratic Casimir of R_i

$Y_{ijk} = (Y_{ijk})^*$

$\Rightarrow$ Yukawa and gauge couplings are related.

Note • μ^{ij} are not restricted

•• Appearance of $U(1)$ is incopatible with 1^{st} condt.

••• 2^{nd} condt forbids the presence of singlets with nonvanishing couplings.

$\therefore \Rightarrow$ ~~Susy~~ by G-invariant soft terms

* 1-loop finiteness condts necessary and sufficient to guarantee 2-loop finiteness

* 1-loop finiteness condts ensure that $b_g^{(3)}$ in 3-loops vanishes but in general $\gamma^{(3)}$ does not.

Grisaru - Milewski - Zanon

Parkes - West

What happens in higher loops?

So far 1-loop finiteness condts (on γs) are telling us

$$Y^{ijk} = Y^{ijk}(g)$$

$$b_Y^{(1)ijk} = 0$$

** Necessary and sufficient condts for vanishing b_g and b_{ijk} to all orders

Lucchesi
Piguet
Sibold

1. $b_g^{(1)} = 0$

2. $\gamma^{(1)i}_{\;\;j} = 0$

3. $b_Y^{ijk} = b_g \dfrac{d Y^{ijk}}{dg}$

admit power series solution which in lowest order is a solution of condt 2.

3. → 3'. There exist solutions to $\gamma^{(1)i}_{\;\;j} = 0$ of the form $Y^{ijk} = \rho^{ijk} g$, ρ^{ijk} = complex

→ 4. These solutions are isolated and non-degenerate considered as solutions of $b_Y^{(1)ijk} = 0$

Recall

R-invariance, axial anomaly

In massless $N=1$ ths

$U(1)$ chiral transformation R:

$A_\mu \to A_\mu$, $\lambda \to e^{-i\alpha}\lambda$,

$\phi \to e^{-i\frac{2}{3}\alpha}\phi$, $\psi \to e^{i\frac{1}{3}\alpha}\psi$, ...

$\psi_D = \begin{pmatrix} \psi \\ \bar{\chi} \end{pmatrix} \longrightarrow e^{i\alpha\gamma_5}\psi_D$

Noether current $J^\mu_R = \bar{\lambda}_D \gamma^\mu \gamma^5 \lambda_D + \cdots$

$\rightsquigarrow \partial_\mu J^\mu_R = r\left(\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \cdots\right)$

$r = b_g^{(1)}$!

Only 1-loop contributions due to Adler-Bardeen non-renormalization thm

Supercurrent

$$\mathcal{J} \equiv \{ J'^{\mu}_{R} , Q^{\mu}_{\alpha} , T^{\mu}_{\nu} \} , \cdots$$ vector super multiplet

associated to R-invariance — associated to susy — associated to translation inv.

Ferrara + Zumino

(supercurrent is represented as vector superfield

$$V_\mu(x,\theta,\bar\theta) = R_\mu(x) - i\theta^\alpha Q_{\mu\alpha}(x) + i\bar\theta_{\dot\alpha}\bar Q^{\dot\alpha}_\mu(x) - 2(\theta\sigma^\nu\bar\theta)T_{\mu\nu}(x) + \cdots$$)

- $J'^{\mu}_{R} \neq J^{\mu}_{R}$

·· $J'^{\mu}_{R} = J^{\mu}_{R} + O(\hbar)$

In addition — Clark, Piguet, Sibold

$$S = \{ \underbrace{b_g F^{\mu\nu}F_{\mu\nu} + \cdots}_{\text{trace anomaly of } T^\mu_\nu} , \underbrace{b_g \epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \cdots}_{\text{anomaly of R-current}} ,$$

$$\underbrace{b_g \lambda^\alpha \sigma^{\mu\nu}_{\alpha\beta} F_{\mu\nu} + \cdots}_{\text{trace anomaly of susy current}} , \cdots \}$$ chiral super multiplet

Super-trace anomaly

There is a relation, whose structure is independent from the renormalization scheme, although individual coefficients (except the 1-loop values of b-functions) may be scheme dependent

$$r = b_g(1 + x_g) + b_{ijk}\, x^{ijk} - \gamma_A\, r^A$$

radiative corrections

linear combinations of anomalous dims

unrenormalized coefficients of anomalies associated to chiral inv. of superpotential

Thm: If (i) no gauge anomaly
(ii) $b^{(1)}(g) = 0$ i.e. no R-current anomaly
(iii) $\gamma^{(1)}{}^i_j = 0$ implies also $r^A = 0$
(iv) exist solutions to $\gamma^{(1)} = 0$ of the form $C_{ijk} = \rho_{ijk}\, g$, ρ_{ijk} - complex
(v) these solutions are isolated + non-degenerate

when considered as solutions of $B^{(1)}_{ijk} = 0$.

- Then each of the solutions can be uniquely extended to a formal power series in g, and the $N=1$ Y-M models depend on the single coupling constant g with a b-function vanishing to all orders.

Proof: Inserting $b_{ijk} = b_g \dfrac{d f_{ijk}}{dg}$ in the identity and taking into account the vanishing of r, r^A

$$\rightsquigarrow \quad 0 = b_g \left(1 + O(\hbar)\right)$$

Its solution (as formal power series in $\hbar$) is: $b_g = 0$

and $b_{ijk} = 0$ too. //

2-loop RGEs for SSB parameters

Martin-Vaughn - Yamada - Jack - Jones '94

Consider $N=1$ gauge thy with

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

and SSB terms

$$-\mathcal{L}_{SB} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j$$

$$+ \frac{1}{2} (m^2)^i_j \phi^{*i} \phi_j + \frac{1}{2} M \lambda\lambda + h.c.$$

- 1-loop finiteness conditions

$$h^{ijk} = -M Y^{ijk}$$

$$(m^2)^i_j = \frac{1}{3} M M^* \delta^i_j \quad \text{universality}$$

in addition to $\beta_g^{(1)} = \gamma^{(1)i}_j = 0$

- • 1-loop finiteness

$\rightsquigarrow$ 2-loop finiteness

Assuming

- $b_g^{(1)} = \gamma^{(1)j}_i = 0$
- the reduction eq

$$b_Y^{ijk} = b_g \, dY^{ijk}/dg$$

admits power series solution

$$Y^{ijk} = g \sum_{n=0} \rho_{(n)}^{ijk} g^{2n}$$

- $(m^2)^i_j = m_j^2 \delta^i_j$

$$\rightsquigarrow (m_i^2 + m_j^2 + m_k^2)/MM^* = 1 + \frac{g^2}{16\pi^2}\Delta^{(1)}$$

for i,j,k with $\rho_{(0)}^{ijk} \neq 0$

where $\Delta^{(1)} = -2\sum_l \left[(m_l^2/MM^*) - \frac{1}{3}\right] \ell(h_l)$

- $\Delta^{(1)} = 0$ for $N=4$ with STr cond
- $\Delta^{(1)} = 0$ for the $N=1$, $SU(5)$ FUTs!

$$\mathcal{L}_{N=1} = \int d^2\theta \frac{1}{4g^2} \mathrm{Tr}\, W^\alpha W_\alpha + \int d^2\bar{\theta} \frac{1}{4g^2} \mathrm{Tr}\, \bar{W}^\alpha \bar{W}_\alpha$$

$$+ \int d^2\theta\, d^2\bar{\theta}\, \bar{\Phi}^i \left(e^V\right)_i^j \Phi_j + \int d^2\theta\, W + \int d^2\bar{\theta}\, \bar{W}$$

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

$$\mathcal{L}_{SB} = \frac{1}{2} M \lambda\lambda + \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j$$

$$+ \frac{1}{2} (m^2)^i_j\, \phi^{*i} \phi_j + \mathrm{h.c.}$$

We can rewrite $\mathcal{L}_{SB}$ in terms of $N=1$ superfields introducing external spurion superfields

$$\eta = \theta^2, \quad \bar{\eta} = \bar{\theta}^2$$

$$\mathcal{L}_{N=1} = \int d^2\theta \, \frac{1}{4g^2}\left(1 - 2M\theta^2\right) \mathrm{Tr}\, W^\alpha W_\alpha$$

$$+ \int d^2\bar\theta \, \frac{1}{4g^2}\left(1 - 2\bar M \bar\theta^2\right) \mathrm{Tr}\, \bar W^\alpha \bar W_\alpha$$

$$+ \int d^2\theta \, d^2\bar\theta \; \bar\Phi \left(\delta_i^{\,k} - (m^2)_i^{\,k} \, \eta\bar\eta\right) (e^V)_k^{\,j} \Phi_j$$

$$+ \int d^2\theta \Big[\frac{1}{6}\left(Y^{ijk} - h^{ijk}\eta\right) \Phi_i \Phi_j \Phi_k$$

$$+ \frac{1}{2}\left(\mu^{ij} - b^{ij}\right) \Phi_i \Phi_j \Big] + \text{h.c.}$$

If an $N=1$ thy is renormalised introducing Z_i, then the SB thy is renormalised by $\tilde Z_i$ with

$$\tilde Z_i(g^2, Y) = Z_i(\tilde g^2, \tilde Y), \text{ where}$$

$$\tilde g^2 = g^2\left(1 + M\eta + \bar M \bar\eta + 2M\bar M \eta\bar\eta\right)$$

$$\tilde Y^{ijk} = Y^{ijk} - h^{ijk}\eta + \frac{1}{2}\Big(Y^{njk}(m^2)_n^{\,i} + Y^{ink}(m^2)_n^{\,j}$$

$$+ Y^{ijn}(m^2)_n^{\,k}\Big)\eta\bar\eta$$

The β-functions of M, h, m^2 are

$$\beta_M = 2\,\mathcal{O}\left(\frac{\beta_g}{g}\right)$$

$$\beta_h^{ijk} = \gamma_e{}^i h^{ljk} + \gamma_e{}^j h^{ilk} + \gamma_e{}^k h^{ijl}$$
$$-2\gamma_1{}^i{}_e Y^{ljk} - 2\gamma_1{}^j Y^{ilk} - 2\gamma_1{}^k{}_e Y^{ijl}$$

$$(\beta_{m^2})^i{}_j = \left[\Delta + X\frac{\partial}{\partial g}\right]\gamma^i{}_j$$

$$\mathcal{O} = \left(Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial Y^{lmn}}\right)$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2\frac{\partial}{\partial g^2}$$
$$+ \hat{Y}_{lmn}\frac{\partial}{\partial Y_{lmn}} + \hat{Y}^{lmn}\frac{\partial}{\partial Y^{lmn}}$$

$$(\gamma_1)^i{}_j = \mathcal{O}\,\gamma^i{}_j \quad , \quad Y_{lmn} = (Y^{lmn})^*$$
$$\hat{Y}^{ijk} = (m^2)^i{}_e Y^{ljk} + (m^2)^j{}_e Y^{ilk} + (m^2)^k{}_e Y^{ijl}$$

Also

$$\beta_b^{ij} = \gamma^i{}_e b^{lj} + \gamma^j{}_e b^{il}$$
$$-2\gamma_1{}^i{}_e \mu^{lj} - 2\gamma_1{}^j{}_e \mu^{il}$$

For simplicity assume the case of a single real Y and a single Φ in a simple gauge group.

Assuming that

- $\beta_g^{(1)} = \gamma^{(1)} = 0$

- the reduction eq

$$\beta_Y = \beta_g \frac{dY}{dg}$$

admits power series solution

$$Y = c_1 g + c_2 g^3 + \cdots$$

$\rightsquigarrow$ $N=1$ finite thy to all orders

In the SB thy we search for a RGI surface on which O and Δ become total derivative terms.

e.g. inspection of

$$O = \frac{1}{2}\left(M g \frac{\partial}{\partial g} - h \frac{\partial}{\partial Y} \right)$$

suggests to demand

$$h = - M g \frac{dY}{dg}$$

$$\rightsquigarrow \quad O = \frac{1}{2} M g \frac{d}{dg}$$

In this case since

$$\gamma(g, Y(g)) = 0 \quad \text{for any } g$$

$$\rightsquigarrow \quad \gamma_1 = O\gamma = 0$$

$$\Longrightarrow \quad \beta_h = 0 \quad \text{to all orders}$$

Note that in the approximation

$$Y = c_1 g$$

$$\rightsquigarrow \quad h \equiv - M Y \quad \text{i.e. condition for 1+2 loop finiteness}$$

In addition

$\Longrightarrow$ $b_M = 0$; $b_b = 0$

to all orders

- The b_{m^2} is a little more complicated.

Requiring a more complicated expression to be RGI

$$\rightsquigarrow \Delta = |M|^2\left[\frac{1}{2}\frac{d^2}{d(\ln g)^2} + \left(1 + \tilde{X}(g)/g\right)\frac{d}{d\ln g}\right]$$

$\Longrightarrow$ $b_{m^2} = 0$ to all orders

- The general $b_{m_i^2}^{NSVZ}$ is given by

$$b_{m_i}^{NSVZ} = \left[|M|^2\left\{\frac{1}{1-g^2C(G)/8\pi^2}\frac{d}{dg} + \frac{1}{2}\frac{d^2}{d(\ln g)^2}\right\} + \sum_\ell \frac{m_\ell^2\, T(R_\ell)}{C(G)-8\pi^2/g^2}\frac{d}{d\ln g}\right]\gamma_i^{NSVZ}$$

~~Kobayashi~~ Kubo

Z

to all orders

In addition holds to all orders that

$$m_i^2 + m_j^2 + m_k^2$$

$$= |M|^2 \left\{ \frac{1}{1 - g^2 C(G)/8\pi^2} \frac{d \ln Y^{ijk}}{d \ln g} + \frac{1}{2} \frac{d^2 \ln Y^{ijk}}{d(\ln g)^2} \right\}$$

$$+ \sum_\ell \frac{m_\ell^2 T(R_\ell)}{C(G) - 8\pi^2/g^2} \frac{d \ln Y^{ijk}}{d \ln g}$$

which in the finite case in 2-loops becomes

$$m_i^2 + m_j^2 + m_k^2 = |M|^2 \left\{ 1 + \frac{g^2}{16\pi^2} \Delta^{(1)} \right\}$$

$$\Delta^{(1)} = -2 \sum_\ell \left[m_\ell^2/|M|^2 - (1/3) \right] T(R_\ell)$$

$\Delta^{(1)}$ vanishes for $m_i^2 = m_j^2 = m_k^2$

i.e. $m^2 = \frac{1}{3} |M|^2$ up to 2-loops

for the universal choice.

Also in general (non-finite) $N=1$ thies

$$b_M = 2\,\mathcal{O}\left(\frac{b_g}{g}\right)$$

with $\mathcal{O} = \frac{M}{2} g \frac{d}{dg}$

Searching for

$$f(g)\, M = RGI$$

i.e. $\frac{\partial f(g)}{\partial g} b(g)\, M + f(g)\, b_M = 0$

$$\rightsquigarrow \quad M \frac{g}{b(g)} = RGI$$

Hisano-Shifman

An interesting remark   Kazakov

$$\hat{g} = g\left(1 + M\eta + \bar{M}\bar{\eta} + 2M\bar{M}\eta\bar{\eta}\right)$$

$$\tilde{Y} = Y\left(1 - h\eta - \bar{h}\bar{\eta} + \cdots\right)$$

We require that

$$\tilde{Y} = Y(\hat{g})$$

and expand over $\eta, \bar{\eta}$

$$\rightsquigarrow \tilde{Y} = Y(\hat{g}) = Y(g) + \frac{dY}{dg} g M \eta + \cdots$$

$$\rightsquigarrow -Yh = \frac{dY}{dg} g M$$

$$\rightsquigarrow h = -M \frac{d\ln Y}{d\ln g}$$

Anomaly-mediated ~~SUSY~~

$$\Rightarrow \begin{cases} M = m_{3/2}\, \beta_g / g \\ h^{ijk} = -m_{3/2}\, \beta_C^{ijk} \\ b^{ij} = -m_{3/2}\, \beta_\mu^{ij} \\ (m^2)^i_j = \frac{1}{2} |m_{3/2}|^2 \frac{d\gamma^i_j}{dt} \end{cases}$$

RGI to all-loops

Assuming

existence of RGI surfaces on which

a) $C = C(g)$ or

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g}$$

b) $$h^{ijk} = -M \frac{dC(g)^{ijk}}{d\ln g}$$

without relying on specific solutions

$\rightsquigarrow$ consequences of anomaly-med. susy scenario are obtained from the b-functions of SSB parameters.

Assuming $c\frac{\partial}{\partial c} = c^* \frac{\partial}{\partial c^*}$

for a RGI surface $F(g, C^{ijk}, C^{*ijk})$

$$\rightsquigarrow \frac{d}{dg} = \left(\frac{\partial}{\partial g} + 2\,\frac{b_c}{b_g}\,\frac{\partial}{\partial c}\right) \Big|\Big|$$

- Consider

$$\mathcal{O} = \left(M g^2 \frac{\partial}{\partial g^2} - h \frac{\partial}{\partial c}\right)$$

$$\overset{(b)}{\rightsquigarrow} \quad \mathcal{O} = \frac{1}{2} M \frac{d}{d\ln g}$$

and $$b_M = M \frac{d}{d\ln g}\left(\frac{b_g}{g}\right)$$

$$F(g, c, c^*) = \text{const}$$

$$dF = \left(\frac{\partial}{\partial g}\, dg + \frac{\partial}{\partial c}\, dc + \frac{\partial}{\partial c^*}\, dc^*\right) F$$

$$= 0$$

$$\rightsquigarrow \frac{dF}{dg} = \left(\frac{\partial}{\partial g} + \frac{dc}{dg}\frac{\partial}{\partial c} + \frac{dc^*}{dg}\frac{\partial}{\partial c^*}\right) F$$

$$= 0$$

and if $c\dfrac{\partial}{\partial c} = c^*\dfrac{\partial}{\partial c^*}$

$$\rightsquigarrow \frac{dF}{dg} = \left(\frac{\partial}{\partial g} + 2\frac{\partial}{\partial c}\frac{dc}{dg}\right) F = 0$$

$$\rightsquigarrow \frac{d}{dg} = \frac{\partial}{\partial g} + 2\frac{\beta_c}{\beta_g}\frac{\partial}{\partial c} \qquad \Big\Vert$$

$\rightsquigarrow M = \frac{b_g}{g} M_0$ || Generalized Hisano-Shifman

M_0 - integration const. which in sugra becomes $m_{3/2}$

$\rightsquigarrow b_M = m_{3/2} \frac{d}{dt}\left(b_g/g\right)$

∴ Similarly

$$(\gamma_1)^i_j = \mathcal{O}\gamma^i_j = \frac{1}{2} m_{3/2} \frac{d\gamma^i_j}{dt} \quad \|$$

∴ From (b) and H-S

$\rightsquigarrow h^{ijk} = - m_{3/2} \, b_c^{ijk}$

and using $(\gamma_1)^i_j$ above

$\rightsquigarrow b_h^{ijk} = - m_{3/2} \frac{d}{dt} b_c^{ijk}$

$\rightsquigarrow h^{ijk} = - m_{3/2} \, b_c^{ijk}$

is RGI

.... We have also proved that the sum rule is also RGI to all loops which generalizes the corresponding relation for $(m^2)^i_j$

Remarks

- Differences in assuming existence of RGI surfaces in (a) + (b) and considering specific solution of REs.
- e.g. at 1st order in g the sum rule in first case

$$m_i^2 + m_j^2 + m_k^2 = |M|^2 \frac{d \ln C^{ijk}}{d \ln g}$$

and $$\frac{d \ln C^{ijk}}{d \ln g} = \frac{g}{C^{ijk}} \frac{d C^{ijk}}{d g} = \frac{g}{C^{ijk}} \frac{\beta_C^{ijk}}{\beta_g}$$

which is clearly model dependent.

but assuming a power series solution

$$\frac{d\, C^{ijk}}{d \ln g} = 1$$

model independent!

•• All-loop sum rule does not depend on specific solution while

$$(m^2)^i{}_j = \frac{1}{2} \frac{g^2}{b_g} |M|^2 \frac{d \gamma^i{}_j}{d g}$$

it does!

••• Resolution of the fatal problem of anomaly induced scenario:

Use the Sum Rule!

The SU(5) finite model

Kapetanakis, Mondragon, Z

Kobayashi, Kubo, Mondragon, Z

Content

Hamidi-Schwarz
Jones-Raby
Quiros et. al.
Kazakov

$$3(\bar{5}+10)+4(\overset{H_\alpha\ \bar{H}_\alpha}{5+\bar{5}})+24$$

fermion supermultiplets

scalar supermultiplets

Imposing a discrete symmetry

$$\Rightarrow W=\sum_{i=1}^{3}\sum_{\alpha=1}^{4}\left[\frac{1}{2}g^u_{i\alpha}\,10_i\,10_i\,H_\alpha + g^d_{i\alpha}\,10_i\,\bar{5}_i\,\bar{H}_\alpha\right]+\sum_{\alpha=1}^{4}g^f_\alpha\,H_\alpha\,24\,\bar{H}_\alpha + \frac{g^\lambda}{3}(24)^3$$

with $g^{u,d}_{i\alpha}=0$ for $i\neq\alpha$

We find

$$b_g^{(1)} = 0$$

$$b_{i\alpha}^{u(1)} = \frac{1}{16\pi^2}\left[-\frac{96}{5}g^2 + \sum_{b=1}^{4}\left(g_{ib}^{u}\right)^2 + 3\sum_{j=1}^{3}\left(g_{ja}^{u}\right)^2 + \frac{24}{5}\left(g_{\alpha}^{f}\right)^2 + 4\sum_{b=1}^{4}\left(g_{ib}^{d}\right)^2\right]g_{i\alpha}^{u}$$

$$b_{i\alpha}^{d(1)} = \frac{1}{16\pi^2}\left[-\frac{84}{5}g^2 + 3\sum_{b=1}^{4}\left(g_{ib}^{u}\right)^2 + \frac{24}{5}\left(g_{\alpha}^{f}\right)^2 + 4\sum_{j=1}^{3}\left(g_{j\alpha}^{d}\right)^2 + 6\sum_{b=1}^{4}\left(g_{ib}^{d}\right)^2\right]g_{i\alpha}^{d}$$

$$b^{\lambda(1)} = \frac{1}{16\pi^2}\left[-30g^2 + \frac{63}{5}\left(g^{\lambda}\right)^2 + 3\sum_{\alpha=1}^{4}\left(g_{\alpha}^{f}\right)^2\right]g^{\lambda}$$

$$b_{\alpha}^{f(1)} = \frac{1}{16\pi^2}\left[-\frac{98}{5}g^2 + 3\sum_{i=1}^{3}\left(g_{i\alpha}^{u}\right)^2 + 4\sum_{i=1}^{3}\left(g_{i\alpha}^{d}\right)^2 + \frac{48}{5}\left(g_{\alpha}^{f}\right)^2 + \sum_{b=1}^{4}\left(g_{b}^{f}\right)^2 + \frac{21}{5}\left(g^{\lambda}\right)^2\right]g_{\alpha}^{f}$$

Considering g as the primary coupling, we solve the Red. Eqs.

$$\beta_g = \beta_a \frac{dg}{dg_a}$$

requiring power series ansatz.

$$\rightsquigarrow (g^u_{ii})^2 = \frac{8}{5} g^2 + \cdots , (g^d_{ii})^2 = \frac{6}{5} g^2 + \cdots$$

$$(g^\lambda)^2 = \frac{15}{7} g^2 + \cdots , (g^f_4)^2 = g^2 , (g^f_\alpha)^2 = 0 + \cdots (\alpha = 1,2,3)$$

Higher order terms can be uniquely determined.

$\Longrightarrow$ All 1-loop β-functions vanish

Moreover

All 1-loop anomalous dimensions of chiral fields vanish.

$$\gamma^{(1)}_{10i} = \frac{1}{16\pi^2}\left[-\frac{36}{5}g^2 + 3\sum_{b=1}^{4}\left(g^u_{ib}\right)^2 + 2\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\right]$$

$$\gamma^{(1)}_{\bar{5}i} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 4\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\right]$$

$$\gamma^{(1)}_{H\alpha} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 3\sum_{i=1}^{3}\left(g^u_{i\alpha}\right)^2 + \frac{24}{5}\left(g^f_{\alpha}\right)^2\right]$$

$$\gamma^{(1)}_{\bar{H}\alpha} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 4\sum_{i=1}^{3}\left(g^d_{i\alpha}\right)^2 + \frac{24}{5}\left(g^f_{\alpha}\right)^2\right]$$

$$\gamma^{(1)}_{24} = \frac{1}{16\pi^2}\left[-\frac{10}{5}g^2 + \sum_{\alpha=1}^{4}\left(g^f_{\alpha}\right)^2 + \frac{21}{5}\left(g^\lambda\right)^2\right]$$

$\Longrightarrow$ Necessary and sufficient cond. for finiteness to all orders are satisfied

- SU(5) breaks down to the standard model due to $\langle 24 \rangle$

- Use the freedom in mass parameters to obtain only a pair of Higgs fields light, acquiring v.e.v.

- Get rid of unwanted triplets rotating the Higgs sector (after a fine tuning)

  see Quiros et. al., Kazakov et. al
  Yioshioka

- Adding soft terms we can achieve SUSY breaking.

1) Gauge Couplings Unification

$\sin^2\theta_W, \alpha_{em} \rightsquigarrow \alpha_3(M_Z)$

Marciano + Senjanović
Amaldi et. al.

2) Bottom-Tau Yukawa Unif.

SU(5)-type

$\rightsquigarrow m_t \sim 100-200$ GeV

Barger et. al
Carena et. al.

*3) Top-Bottom-Tau Yuk Unif.

$h_t^2 = \frac{4}{3} h_{b,\tau}^2$ in SU(5)-FI?

Similar to SO(10)

Ananthanarayan et. al.
Barger et. al.

$\rightsquigarrow m_t \sim 160-200$ GeV

Carena et. al.

✱4) Gauge-Top-Bottom-Tau Unif.

e.g. FI?-SU(5): $h_t^2 = \frac{8}{5} g_U^2$; $h_{b,\tau}^2 = \frac{6}{5} g_U^2$

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	M_{GUT} [GeV]	M_b [GeV]	M_t [GeV]
300	0.123	54.1	2.2×10^{16}	5.3	183
500	0.122	54.2	1.9×10^{16}	5.3	183
10^3	0.120	54.3	1.5×10^{16}	5.2	184

FUTA

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	M_{GUT} [GeV]	M_b [GeV]	M_t [GeV]
800	0.120	48.2	1.5×10^{16}	5.4	174
10^3	0.119	48.2	1.4×10^{16}	5.4	174
1.2×10^3	0.118	48.2	1.3×10^{16}	5.4	174

FUTB

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	M_{GUT} [GeV]	M_b [GeV]	M_t [GeV]
300	0.123	47.9	2.2×10^{16}	5.5	178
500	0.122	47.8	1.8×10^{16}	5.4	178
1000	0.119	47.7	1.5×10^{16}	5.4	178

MIN SU(5)

The predictions for the three models for different M_s

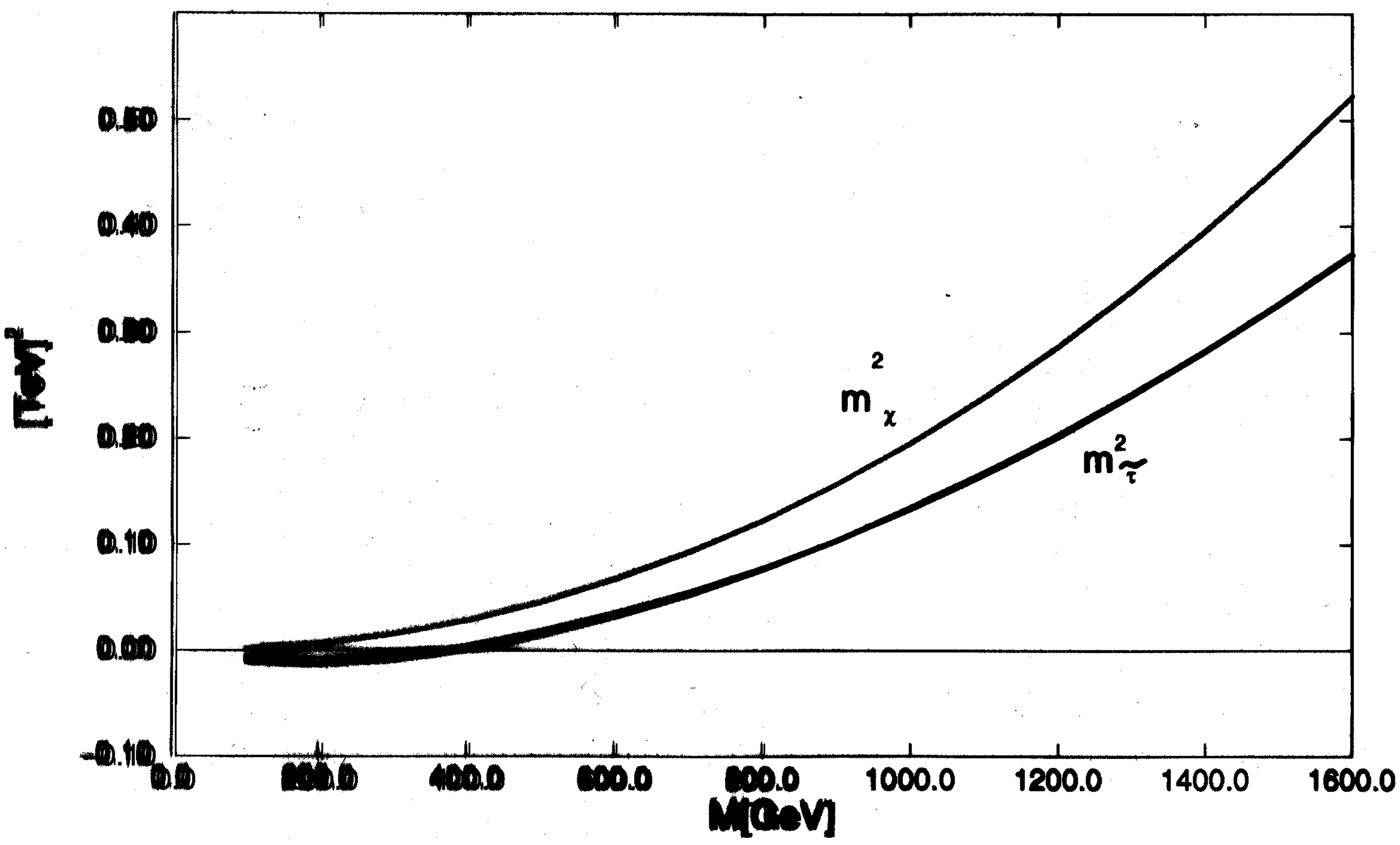
With theoretical corrections and uncertainties[8]

$\sim 4\%$

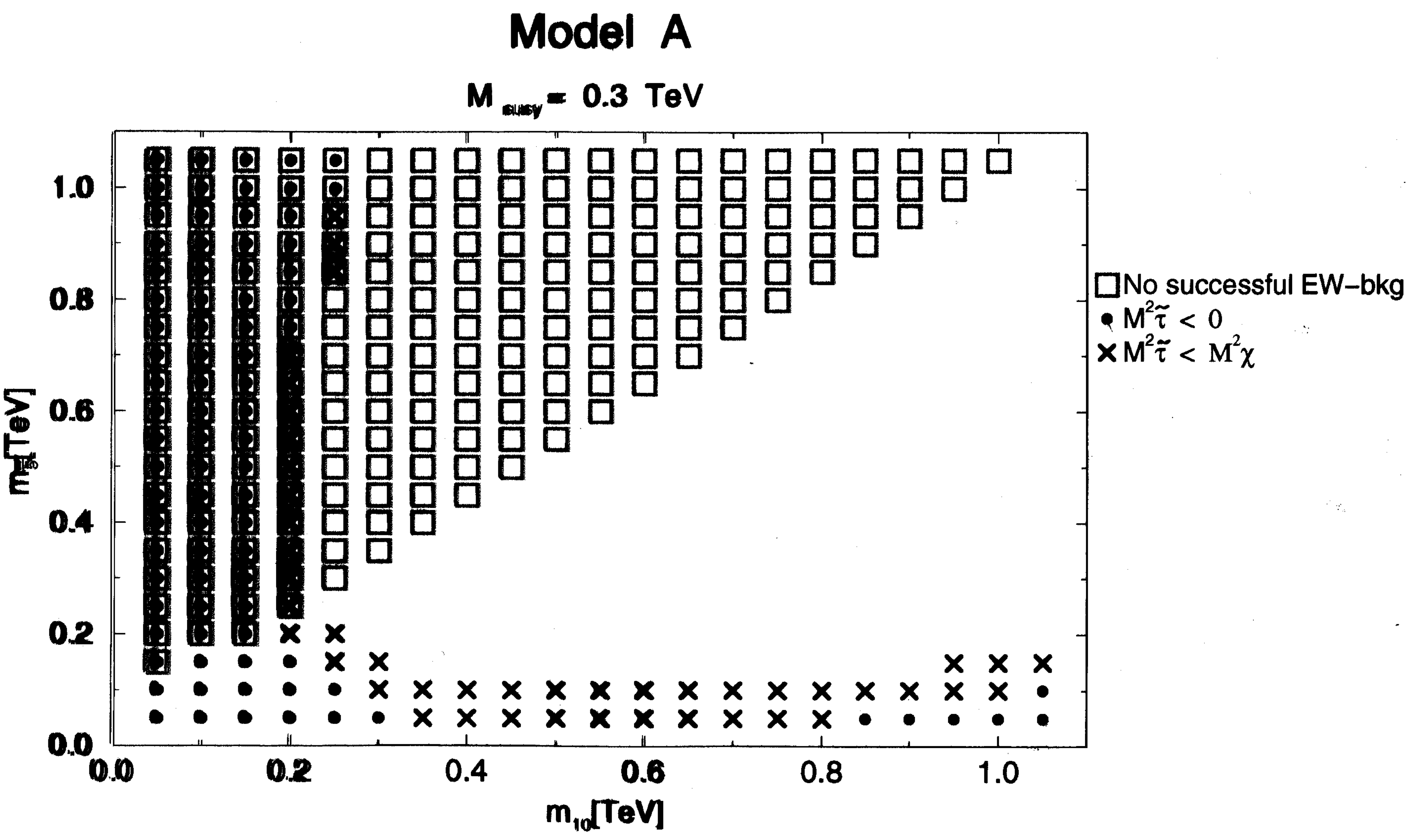
$M_t = 173.8 \pm 5$ GeV

Model A

Similar behaviour holds for Model B too



$m^2_{\tilde{\tau}}$ and m^2_χ for the universal choice of soft scalar masses



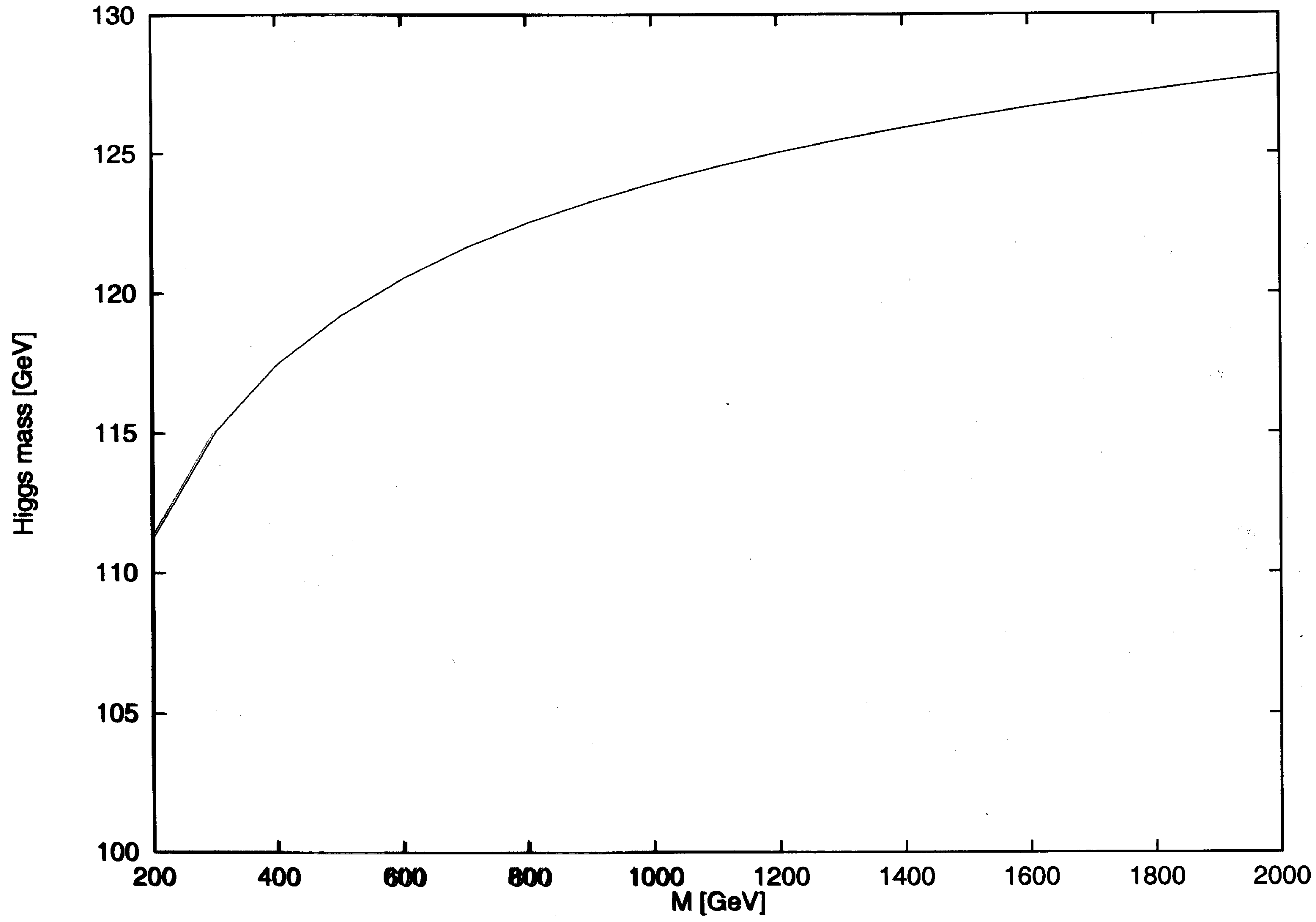
The empty region yields a neutralino as LSP

A representative example of the predictions for the s-spectrum for the finite model **A** with $M = 1\text{TeV}$, $m_{\overline{5}} = 0.7$ TeV and $m_{10} = 0.7$ TeV.

$m_\chi = m_{\chi_1}$	(TeV)	0.44	$m_{\tilde{b}_2}$	(TeV)	1.66
m_{χ_2}	(TeV)	0.84	$m_{\tilde{\tau}} = m_{\tilde{\tau}_1}$	(TeV)	0.53
m_{χ_3}	(TeV)	1.46	$m_{\tilde{\tau}_2}$	(TeV)	0.92
m_{χ_4}	(TeV)	1.46	$m_{\tilde{\nu}_1}$	(TeV)	0.90
$m_{\chi_1^\pm}$	(TeV)	0.84	m_A	(TeV)	0.41
$m_{\chi_2^\pm}$	(TeV)	1.46	$m_{H^\pm}$	(TeV)	0.42
$m_{\tilde{t}_1}$	(TeV)	1.67	m_H	(TeV)	0.41
$m_{\tilde{t}_2}$	(TeV)	1.82	m_h	(TeV)	0.122
$m_{\tilde{b}_1}$	(TeV)	0.86			

A representative example of the predictions of the s-spectrum for the finite model **B** with $M = 1$ TeV and $m_{10} = 0.65$ TeV.

$m_\chi = m_{\chi_1}$	(TeV)	0.44	$m_{\tilde{b}_2}$	(TeV)	1.79
m_{χ_2}	(TeV)	0.84	$m_{\tilde{\tau}} = m_{\tilde{\tau}_1}$	(TeV)	0.47
m_{χ_3}	(TeV)	1.38	$m_{\tilde{\tau}_2}$	(TeV)	0.69
m_{χ_4}	(TeV)	1.39	$m_{\tilde{\nu}_1}$	(TeV)	0.62
$m_{\chi_1^\pm}$	(TeV)	0.84	m_A	(TeV)	0.74
$m_{\chi_2^\pm}$	(TeV)	1.39	$m_{H^\pm}$	(TeV)	0.75
$m_{\tilde{t}_1}$	(TeV)	1.60	m_H	(TeV)	0.74
$m_{\tilde{t}_2}$	(TeV)	1.82	m_h	(TeV)	0.117
$m_{\tilde{b}_1}$	(TeV)	1.56			



Higgs mass [GeV]
130
125
120
115
110
105
100
200
400
600
800
1000
1200
1400
1600
1800
2000
M [GeV]