6-dimensional super-3-brane action

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Algorithm of constructing super-p-brane action in D dim:

1) supersymmetry algebra
2) linear representation of the algebra
3) construction of finite supersymmetry transformations and replacing parameters by Goldstone fields
4) putting the finite supersymmetry transformations to be equal to zero
5) solving obtained equations

Example. $N=1$ $D=4$ supermembrane

$N=1$ $D=4$ supermembrane breaks half of the $N=1$ $D=4$ supersym. and $D-p-1=4-2-1=1$ translation.

1) Algebra

$N=1$ $D=4$ SUSY is equivalent to $N=2$ $d=3$ SUSY with one central charge

$$\{Q_\alpha, Q_\beta\} = P_{\alpha\beta} \qquad \{S_\alpha, S_\beta\} = P_{\alpha\beta} \qquad \{Q_\alpha, S_\beta\} = \varepsilon_{\alpha\beta} Z$$

Q_α, S_α – supersymmetry generators $\qquad \alpha, \beta, ... = 1,2$

$P_{\alpha\beta} = i\partial_{\alpha\beta} = i\sigma^m_{\alpha\beta}\partial_m$ – translation generators $\qquad m = 0,1,2$

Z – central charge generator

Broken symmetry generators: S_α, Z

Unbroken ——"——"——: Q_α, $P_{\alpha\beta}$

2) Representation

$$\delta\varphi(x,\theta) = i\eta^\alpha \xi_\alpha$$
$$\delta\xi_\alpha(x,\theta) = \eta_\alpha\left(1-\frac{i}{2}\mathcal{D}^2\varphi\right) - \frac{i}{2}\eta^\beta\partial_{\alpha\beta}\varphi$$

η_α – S-supersymmetry parameters

θ^α – Grassmann coordinates corresp. Q-supersymmetry

$\mathcal{D}_\alpha$ – covariant spinor derivative —"—"——"——: $\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - \frac{i}{2}\theta^\beta\partial_{\alpha\beta}$

$\mathcal{D}^2 \equiv \mathcal{D}^\alpha\mathcal{D}_\alpha$

S-supersymmetry and Z-translation are broken.

ξ_α – Goldstone fermion of linear realization

3) Finite transformations

$$\tilde{\varphi} \equiv \varphi + \delta\varphi + \frac{1}{2!}\delta^2\varphi + \frac{1}{3!}\delta^3\varphi + \dots$$
$$\tilde{\xi}_\alpha \equiv \xi_\alpha + \delta\xi_\alpha + \frac{1}{2!}\delta^2\xi_\alpha + \frac{1}{3!}\delta^3\xi_\alpha + \dots$$

$\delta^2\varphi = \delta(\delta\varphi)$, $\delta^3\varphi = \delta(\delta^2\varphi)$, ...

$$\tilde{\xi}_\alpha(\eta\to-\psi) = \xi_\alpha - \psi_\alpha\left(1-\frac{i}{2}\mathcal{D}^2\varphi\right) + \frac{i}{2}\psi^\beta\partial_{\alpha\beta}\varphi - \frac{1}{4}\psi^2\partial_{\alpha\beta}\xi^\beta$$
$$\tilde{\varphi}(\eta\to-\psi) = \varphi - i\psi^\alpha\xi_\alpha + \frac{i}{2}\psi^2\left(1-\frac{i}{2}\mathcal{D}^2\varphi\right)$$

4) Covariant constraints:

$$\tilde{\xi}_\alpha(\eta\to-\psi) = 0$$
$$\tilde{\varphi}(\eta\to-\psi) = 0$$

5) Solving the system of equations

$$\begin{cases} \xi_\alpha - \psi_\alpha \left(1 - \frac{i}{2} D^2 \varphi\right) + \frac{i}{2} \psi^\beta \partial_{\alpha\beta} \varphi - \frac{1}{4} \psi^2 \partial_{\alpha\beta} \xi^\beta = 0 \\ \varphi - i \psi^\alpha \xi_\alpha + \frac{i}{2} \psi^2 \left(1 - \frac{i}{2} D^2 \varphi\right) = 0 \end{cases}$$

Solution

$$\psi_\alpha = \frac{\xi_\alpha}{1 - \frac{i}{2} D^2 \varphi}$$

$$\varphi = \frac{i \xi^2}{1 + \sqrt{1 + D^2 \xi^2}}$$

Action of $N=1$ $D=4$ supermembrane is

$$S = \int d^3x\, d^2\theta \frac{i \xi^2}{1 + \sqrt{1 + D^2 \xi^2}}$$

$N=1$ $D=6$ super-3-brane

$N=1$ $D=6$ super-3-brane breaks half of the $N=1$ $D=6$ supersymmetries and $D-p-1 = 6-3-1 = 2$ translations.

1) Algebra

$N=1$ $D=6$ SUSY is equivalent to $N=2$ $D=4$ SUSY with two central charges

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = -2i \partial_{\alpha\dot\alpha} \qquad \{S_\alpha, \bar{S}_{\dot\alpha}\} = -2i \partial_{\alpha\dot\alpha}$$

$$\{Q_\alpha, S_\beta\} = 2 \varepsilon_{\alpha\beta} Z \qquad \{\bar{Q}_{\dot\alpha}, \bar{S}_{\dot\beta}\} = -2 \varepsilon_{\dot\alpha\dot\beta} \bar{Z}$$

$Q_\alpha, \bar{Q}_{\dot\alpha}, S_\alpha, \bar{S}_{\dot\alpha}$ – supersymmetry generators $\qquad \alpha, \dot\alpha, \beta, \dot\beta, \ldots = 1,2$

$Z, \bar{Z}$ – complex central charges $\qquad m = 0,1,2,3$

$\partial_{\alpha\dot\alpha} = \sigma^m_{\alpha\dot\alpha} \partial_m$

Broken symmetry generators: S_α, $\bar{S}_{\dot\alpha}$, Z, $\bar{Z}$

Unbroken symmetry generators: Q_α, $\bar{Q}_{\dot\alpha}$, $P_{\alpha\dot\alpha} \equiv i\partial_{\alpha\dot\alpha}$

2) Representation

$$\delta\varphi = f + \eta^\alpha W_\alpha \qquad\qquad f \equiv c - 2i\eta^\alpha\theta_\alpha$$

$$\delta W_\alpha = -\frac{1}{2}\eta_\alpha \bar{\mathcal{D}}^2\bar\varphi - 2i\bar\eta^{\dot\alpha}\partial_{\alpha\dot\alpha}\varphi$$

c – complex Z-transformation parameter

η^α – Grassmann S-supersymmetry transformation parameter

θ^α – Grassmann coordinate corresp. Q-supersymmetry

$\varphi = \varphi(x, \theta, \bar\theta)$ and $W_\alpha = W_\alpha(x, \theta, \bar\theta)$ are chiral $N=1$ $D=4$ superfields

$$\bar{\mathcal{D}}_{\dot\alpha}\varphi = 0 \qquad\qquad \bar{\mathcal{D}}_{\dot\alpha} W_\alpha = 0$$

Reality condition

$$\mathcal{D}^\alpha W_\alpha + \bar{\mathcal{D}}_{\dot\alpha}\bar{W}^{\dot\alpha} = 0$$

Solution of reality condition equation

$$W_\alpha = \frac{i}{4}\bar{\mathcal{D}}^2\mathcal{D}_\alpha \mathcal{Z}\,, \quad \mathcal{Z} \text{ – real } N=1\ D=4 \text{ superfield}$$

Extended vector supermultiplet

$$\delta\mathcal{Z} = f\bar\varphi + \bar{f}\varphi + \eta^\alpha Z_\alpha + \bar\eta_{\dot\alpha}\bar{Z}^{\dot\alpha}$$

$$\delta Z_\alpha = \bar{f} W_\alpha - \frac{1}{2}\eta_\alpha\bar{\mathcal{D}}^2\bar{F} - \bar\eta^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha\mathcal{Z}$$

$$\delta F = f\varphi + \eta^\alpha Y_\alpha$$

$$\delta Y_\alpha = f W_\alpha - \frac{1}{2}\eta_\alpha\bar{\mathcal{D}}^2\mathcal{Z} - 2i\bar\eta^{\dot\alpha}\partial_{\alpha\dot\alpha}F$$

F, Z_α, Y_α – chiral $N=1$ $D=4$ superfields

3) Finite transformations

There are 2 parameters: $f = c - 2i\eta^{\alpha}\theta_{\alpha}$ – even

η^{α} – odd

$$\tilde{\varphi} = \varphi + \delta\varphi + \frac{1}{2!}\delta^2\varphi + \frac{1}{3!}\delta^3\varphi + \frac{1}{4!}\delta^4\varphi + \frac{1}{5!}\delta^5\varphi$$

$f \to -Q$ – Goldstone boson of nonlinear realization

$\eta_{\alpha} \to -\psi_{\alpha}$ – Goldstone fermion of nonlinear realization

4) Impose covariant constraints

$$\tilde{\varphi}(f \to -Q, \eta \to -\psi) = 0$$
$$\tilde{W}_{\alpha}(f \to -Q, \eta \to -\psi) = 0$$
$$\tilde{\mathcal{Z}}(f \to -Q, \eta \to -\psi) = 0$$
$$\tilde{Z}_{\alpha}(f \to -Q, \eta \to -\psi) = 0$$
$$\tilde{F}(f \to -Q, \eta \to -\psi) = 0$$
$$\tilde{Y}_{\alpha}(f \to -Q, \eta \to -\psi) = 0$$

5) Solution

$$\mathcal{Z} = \varphi\bar{\varphi} + \frac{1}{16}(\mathcal{D}\varphi)^2(\bar{\mathcal{D}}\bar{\varphi})^2 + O(\varphi^6)$$

Action of 6-dimensional super-3-brane

$$S = \int d^4x\, d^2\theta\, d^2\bar{\theta}\, \mathcal{Z} = \int d^4x\, d^2\theta\, d^2\bar{\theta}\left[\varphi\bar{\varphi} + \frac{1}{16}(\mathcal{D}\varphi)^2(\bar{\mathcal{D}}\bar{\varphi})^2 + \dots\right]$$