String/Brane Tension as Dynamical Degree of Freedom

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Based on and further developing:

- E. Guendelman, *Class. Quant. Grav.* **17** (2000) 3673 (*hep-th/0005041*); *Phys. Rev.* **D63** (2001) 046006 (*hep-th/0006079*)
- E. Guendelman, A. Kaganovich, E.N. and S. Pacheva, *Phys. Rev. D*, to appear (*hep-th/0203024*)

Main ideas borrowed from:

- E. Guendelman, *Class. Quant. Grav.* **17** (2000) 261; *Found. Phys.* **31** (2001) 1019
- E. Guendelman and A. Kaganovich, *Phys. Rev.* **D60** (1999) 065004

Related recent developments:

- E. Guendelman and A. Kaganovich, *Int. J. Mod. Phys.* **A17** (2002) 417 (*hep-th/0106152*); *Mod. Phys. Lett.* **A17** (2002) 1227 (*hep-th/0110221*)

Plan of Talk

- Introduction - Main Motivation
- Bosonic Strings with a Modified World-Sheet Integration Measure - Dynamical Generation of String Tension
- Green-Schwarz Superstrings with a Modified World-Sheet Integration Measure
- Application of Modified String Model - Classical Confinement Mechanism of "Color" Charges via Dynamical String Tension
- Branes with a Modified World-Volume Integration Measure - Dynamical Generation of Brane Tension
- Confinement of Charged Lower-Dimensional Branes

Introduction - Main Motivation

Reparametrization-covariant (generally-covariant) integration measure densities:

- Standard Riemannian: $\sqrt{-g}$ with $g \equiv \det \|g_{\mu\nu}\|$
- Modified non-Riemannian:
  $\Phi(\varphi) \equiv \frac{1}{D!}\varepsilon^{\mu_1 \ldots \mu_D}\varepsilon_{i_1 \ldots i_D}\partial_{\mu_1}\varphi^{i_1} \ldots \partial_{\mu_D}\varphi^{i_D}$

Models involving Gravity with modified measure, or both standard *and* modified - *Two-Measure Gravitational Models* :

$$S = \int d^D x\, \Phi(\varphi)\, L_1 + \int d^D x\, \sqrt{-g}\, L_2$$

$$L_{1,2} = e^{\frac{\alpha\phi}{M_P}}[-\frac{1}{\kappa}R(g) - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + (\text{Higgs}) + (\text{fermions})]$$

Crucial role played by the new "geometric" field:
$\zeta(x) \equiv \frac{\Phi(\varphi)}{\sqrt{-g}}$ – determined only through the matter fields locally (*i.e.*, without gravitational interaction).

Two-measure gravity models address various basic problems and provide possible solutions:

- Scale invariance and its dynamical breakdown; Spontaneous generation of dimensionfull fundamental scales;
- Cosmological constant problem;
- Geometric origin of fermionic families.

Bosonic Strings with a Modified World-Sheet Integration Measure

Standard Polyakov-type bosonic string action:

$$S = -T \int d^2\sigma \, \frac{1}{2} \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X)$$

$$(\sigma^0, \sigma^1) \equiv (\tau, \sigma) \quad a, b = 0, 1 \quad \mu, \nu = 0, 1, \ldots, D-1$$

γ_{ab} – world-sheet Riemannian metric, $\gamma = \det ||\gamma_{ab}||$;
T – string tension, a dimensionfull quantity introduced *ad hoc.*

Eqs. of motion w.r.t. γ^{ab} :

$$T_{ab} \equiv \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2} \gamma_{ab} \gamma^{cd} \partial_c X^\mu \partial_d X^\nu \right) G_{\mu\nu}(X) = 0$$

Eqs. of motion w.r.t. X^μ :

$$\frac{1}{\sqrt{-\gamma}} \partial_a \left(\sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu \right) + \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0$$

where $\Gamma^\mu_{\nu\lambda} = \frac{1}{2} G^{\mu\kappa} \left(\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda} \right)$ (affine connection for external metric).

Idea: Replace $\sqrt{-\gamma}$ with a new reparametrization-covariant world-sheet integration measure density $\Phi(\varphi)$

$$\Phi(\varphi) \equiv \frac{1}{2}\varepsilon_{ij}\varepsilon^{ab}\partial_a\varphi^i\partial_b\varphi^j = \varepsilon_{ij}\dot{\varphi}^i\partial_\sigma\varphi^j \ .$$

with two additional world-sheet scalars φ^i $(i = 1, 2)$, *i.e.*, naively $d^2\sigma\,\sqrt{-\gamma} \to d^2\sigma\,\Phi(\varphi) = d\varphi^1 \wedge d\varphi^2$.

However, the naively generalized action

$$S_1 = -\frac{1}{2}\int d^2\sigma\,\Phi(\varphi)\gamma^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}(X)$$

has a problem – eqs. of motion w.r.t. γ^{ab} lead to an unacceptable condition: $\Phi(\varphi)\,\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}(X) = 0$.

Remedy: Consider topological (total-derivative) terms w.r.t. standard Riemannian world-sheet integration measure. Upon measure replacement $\sqrt{-\gamma} \to \Phi(\varphi)$ the former are *not any more* topological – they will contribute nontrivially to the eqs. of motion. For instance:

$$\int d^2\sigma\,\sqrt{-\gamma}\,R \to \int d^2\sigma\,\Phi(\varphi)\,R \quad , \quad R = \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}}\left(\partial_a\omega_b - \partial_b\omega_a\right)$$

where R is scalar curvature w.r.t. $D = 2$ spin-connection $\omega_a^{\bar{a}\bar{b}} = \omega_a\varepsilon^{\bar{a}\bar{b}}$; ω_a behaves as world-sheet Abelian gauge field.

Modified Bosonic String Action:

$$S = -\int d^2\sigma\, \Phi(\varphi)[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A)]$$

where $F_{ab} = \partial_a A_b - \partial_b A_a$. This action is invariant under diffeomorphisms in φ-target space supplemented with a conformal transformation of γ_{ab} – *"Φ-extended Weyl transformation"*:

$$\varphi^i \longrightarrow \varphi'^i = \varphi'^i(\varphi) \quad , \quad \gamma_{ab} \longrightarrow \gamma'_{ab} = J\gamma_{ab}$$

where $J = \det\|\frac{\partial \varphi'^i}{\partial \varphi^j}\|$.

Eqs. of motion w.r.t. φ^i:

$$\varepsilon^{ab}\partial_b\varphi^i\partial_a(\gamma^{cd}\partial_c X^\mu \partial_d X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd}) = 0$$

implying (provided $\Phi(\varphi) \neq 0$) :

$$\gamma^{cd}\partial_c X^\mu \partial_d X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd} = M \; (= \text{const})$$

Eqs. of motion w.r.t. γ^{ab}:

$$T_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \frac{1}{2}\gamma_{ab}\frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd} = 0$$

Both eqs. of motion above yield $M = 0$ and:

$$T_{ab}\Big|_{Pol} \equiv (\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu) G_{\mu\nu}(X) = 0$$

which is the same as in standard Polyakov-type formulation.

Eqs. of motion w.r.t. X^μ :

$$\partial_a \left(\Phi\gamma^{ab}\partial_b X^\mu\right) + \Phi\gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0$$

where $\Gamma^\mu_{\nu\lambda} = \frac{1}{2} G^{\mu\kappa}\left(\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda}\right)$ is the affine connection corresponding to the external space-time metric $G_{\mu\nu}$.

Most importantly, eqs. of motion w.r.t. A_a yield:

$$\varepsilon^{ab}\partial_b\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\right) = 0$$

which can be integrated to yield a *spontaneously induced* string tension:

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = \text{const} \equiv T$$

We may extend the model by putting point-like charges on the world-sheet coupled to the auxiliary gauge field A_a. In the "static" gauge:

$$S = S_{\text{string}} - \sum_i e_i \int d\tau A_0(\tau, \sigma_i)$$

Then the eq. of motion w.r.t. A_0 becomes:

$$\partial_\sigma\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\right) - \sum_i e_i \delta(\sigma - \sigma_i) = 0$$

It resembles a $D = 2$ Gauss law for the *variable* dynamically induced string tension $T \equiv \Phi(\varphi)/\sqrt{-\gamma}$ identified as a world-sheet electric field strenght.

In fact, it follows directly from the explicit form of the modified-measure string action with charge-current j^a on the string:

$$S = \int d^2\sigma\,[\ldots + \frac{1}{2}\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\varepsilon^{ab}F_{ab}(A)] + \int A_a j^a$$

that the canonically conjugated to A_1 momentum, *i.e.*, the world-sheet electric field strenght is indeed:

$$\pi_{A_1} \equiv E = \frac{\partial \mathcal{L}}{\partial \dot{A}_1} = \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$$

and the eqs. of motion w.r.t. auxiliary gauge field A_a look exactly as $D = 2$ Maxwell eqs.:

$$\varepsilon^{ab}\partial_b E + j^a = 0 \quad , \quad E \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$$

Canonical Hamiltonian Treatment

$$S = -\int d^2\sigma\, \Phi(\varphi)[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A)] - \sum_i e_i \int d\tau A_0(\tau, \sigma_i)$$

Introducing canonical momenta:

$$\pi_i^\varphi = -\varepsilon_{ij}\partial_\sigma \varphi^j [\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A)]$$

$$\pi_{A_1} \equiv E = \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \;,\; \mathcal{P}_\mu = -\Phi(\varphi)\left(\gamma^{00}\dot{X}^\nu + \gamma^{01}\partial_\sigma X^\nu\right) G_{\mu\nu}$$

the canonical Hamiltonian $\mathcal{H} = \sum_A \lambda_A \mathcal{F}_A$ turns out to be linear combination of first-class constraints only. Part of the latter resemble the constraints in the ordinary string case $\pi_{\gamma^{ab}} = 0$ and

$$\mathcal{T}_\pm \equiv \frac{1}{4} G^{\mu\nu}(\frac{\mathcal{P}_\mu}{E} \pm G_{\mu\kappa}\partial_\sigma X^\kappa)(\frac{\mathcal{P}_\nu}{E} \pm G_{\nu\lambda}\partial_\sigma X^\lambda) = 0$$

where in the last Virasoro constraints the dynamical string tension E appears instead of the *ad hoc* constant tension.

The rest of the Hamiltonian constraints are $\pi_{A_0} = 0$,

$$\partial_\sigma E - \sum_i e_i \delta(\sigma - \sigma_i) = 0$$

i.e., the $D = 2$ "Gauss law constraint for the dynamical string tension, and constraints involving only the measure-density fields:

$$\partial_\sigma \varphi^i \pi_i^\varphi = 0 \quad , \quad \frac{\pi_2^\varphi}{\partial_\sigma \varphi^1} = 0$$

The last two constraints span a closed Poisson-bracket algebra:

$$\{ \partial_\sigma \varphi^i \pi_i^\varphi(\sigma) \, , \, \partial_{\sigma'} \varphi^i \pi_i^\varphi(\sigma') \} = \\ 2\partial_\sigma \varphi^i \pi_i^\varphi(\sigma) \partial_\sigma \delta(\sigma - \sigma') + \partial_\sigma \left(\partial_\sigma \varphi^i \pi_i^\varphi \right) \delta(\sigma - \sigma')$$

(a centerless Virasoro algebra), and:

$$\{ \partial_\sigma \varphi^i \pi_i^\varphi(\sigma) \, , \, \frac{\pi_2^\varphi}{\partial_\sigma \varphi^1}(\sigma') \} = -\partial_\sigma (\frac{\pi_2^\varphi}{\partial_\sigma \varphi^1}) \delta(\sigma - \sigma') \; .$$

Therefore, they imply that the measure-density scalars φ^i are pure-gauge degrees of freedom.

Non-Abelian Generalization. Notice the following identity in $D = 2$ involving Abelian gauge field A_a:

$$\frac{1}{2\sqrt{-\gamma}}\varepsilon^{ab}F_{ab}(A) = \sqrt{\frac{1}{2}F_{ab}(A)F_{cd}(A)\gamma^{ac}\gamma^{bd}}$$

This suggests the proper extension of the modified-measure bosonic string model by introducing a *non-Abelian* auxiliary gauge field (here we take for simplicity flat external metric $G_{\mu\nu} = \eta_{\mu\nu}$) :

$$\begin{aligned} S = & -\int d^2\sigma\, \Phi(\varphi)[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu - \\ & -\sqrt{\frac{1}{2}\,\mathrm{Tr}(F_{ab}(A)F_{cd}(A))\gamma^{ac}\gamma^{bd}}] \\ = & -\int d^2\sigma\, \Phi(\varphi)[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu - \\ & -\frac{1}{\sqrt{-\gamma}}\sqrt{\mathrm{Tr}(F_{01}(A)F_{01}(A))}] \end{aligned}$$

where $F_{ab}(A) = \partial_a A_b - \partial_b A_c + i[A_a, A_b]$.

The above action is again invariant under the Φ-*extended Weyl (conformal)* symmetry ($\varphi^i \to \varphi'^i = \varphi'^i(\varphi)$, $\gamma_{ab} \longrightarrow \gamma'_{ab} = \gamma_{ab}\,\det\|\frac{\partial\varphi'^i}{\partial\varphi^j}\|$).

Notice that the *"square-root" Yang-Mills* action (with the regular Riemannian-metric integration measure):

$$\int d^2\sigma \sqrt{-\gamma}\sqrt{\frac{1}{2}\mathrm{T}r(F_{ab}(A)F_{cd}(A))\gamma^{ac}\gamma^{bd}}$$
$$= \int d^2\sigma \sqrt{\mathrm{T}r(F_{01}(A)F_{01}(A))}$$

is a "topological" action similarly to the $D = 3$ Chern-Simmons action (*i.e.*, it is metric-independent).

We can also add non-Abelian "color" point-like charges on the world-sheet with current j^a coupled to the non-Abelian auxiliary gauge field A_a :

$$S = -\int d^2\sigma\, \Phi(\varphi)[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu -$$
$$-\frac{1}{\sqrt{-\gamma}}\sqrt{\mathrm{T}r(F_{01}(A)F_{01}(A))}] + \int \mathrm{T}r\,(A_a j^a)$$

where in the "static" gauge :

$$\int \mathrm{T}r\,(A_a j^a) = -\sum_i \mathrm{T}r\, C_i \int d\tau A_0(\tau, \sigma_i)$$

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$$-\frac{1}{\sqrt{-\gamma}}\sqrt{\mathrm{Tr}(F_{01}(A)F_{01}(A))}] + \int \mathrm{Tr}\,(A_a j^a)$$

where in the "static" gauge :

$$\int \mathrm{Tr}\,(A_a j^a) = -\sum_i \mathrm{Tr}\,C_i \int d\tau A_0(\tau,\sigma_i)$$

Eqs. of motion w.r.t. φ^i :

$$\frac{1}{2}\gamma^{cd}\partial_c X^\mu \partial_d X_\mu - \frac{1}{\sqrt{-\gamma}}\sqrt{\mathrm{Tr}(F_{01}F_{01})} = M \ (= \text{const})$$

Eqs. of motion w.r.t. γ^{ab} :

$$T_{ab} \equiv \partial_a X^\mu \partial_b X_\mu - \frac{1}{\sqrt{-\gamma}}\gamma_{ab}\sqrt{\mathrm{Tr}(F_{01}F_{01})} = 0$$

As in the Abelian case the above eqs. imply $M = 0$ and the Polyakov-type eq.:

$$T_{ab}\Big|_{Pol} \equiv (\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu)G_{\mu\nu}(X) = 0$$

Eqs. of motion w.r.t. auxiliary gauge field A_a – again resemble the $D = 2$ non-Abelian YM eqs.:

$$\varepsilon^{ab}\nabla_a \mathcal{E} + j^a = 0$$

where:

$$\nabla_a \mathcal{E} \equiv \partial_a \mathcal{E} + i[A_a, \mathcal{E}] \ , \ \mathcal{E} \equiv \pi_{A_1} \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \frac{F_{01}}{\sqrt{\mathrm{Tr}(F_{01}F_{01})}}$$

Here $\mathcal{E}$ is the non-Abelian electric field-strength – the canonically conjugated momentum π_{A_1} of A_1, whose norm is the dynamical string tension:

$$T \equiv |\mathcal{E}| = \Phi(\varphi)/\sqrt{-\gamma}$$

The eq. for the dynamical string tension following from $\nabla_a \mathcal{E} + \varepsilon_{ab} j^b = 0$ is:

$$\partial_a \Big(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\Big) + \varepsilon_{ab} \frac{\mathrm{Tr}\left(F_{01} j^b\right)}{\sqrt{\mathrm{Tr}\left(F_{01}^2\right)}} = 0$$

In the absence of external charges ($j^a = 0$) :

$$T \equiv \Phi(\varphi)/\sqrt{-\gamma} = \mathrm{const}$$

The X^μ-eqs. of motion $\partial_a \left(\Phi(\varphi)\gamma^{ab}\partial_b X^\mu\right) = 0$ (here $G_{\mu\nu} = \eta_{\mu\nu}$ for simplicity) can be rewritten in the conformal gauge $\sqrt{-\gamma}\gamma^{ab} = \eta^{ab}$ as:

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} \partial^a \partial_a X^\mu - \widetilde{j}^a \partial_a X^\mu = 0 \quad , \quad \widetilde{j}^a \equiv \frac{\mathrm{Tr}\left(F_{01} j^a\right)}{\sqrt{\mathrm{Tr}\left(F_{01}^2\right)}}$$

For static charges $\widetilde{j}^0 = - \sum_i \widetilde{e}_i \delta(\sigma - \sigma_i)$:

$$T \equiv \Phi(\varphi)/\sqrt{-\gamma} = T_0 + \sum_i \widetilde{e}_i \theta(\sigma - \sigma_i) \;\; , \;\; \widetilde{e}_i \equiv \frac{\mathrm{Tr}\left(F_{01} C_i\right)}{\sqrt{\mathrm{Tr}\left(F_{01}^2\right)}} \Big|_{\sigma=\sigma_i}$$

$$T \partial^a \partial_a X^\mu + \Big(\sum_i \widetilde{e}_i \delta(\sigma - \sigma_i)\Big) \partial_\sigma X^\mu = 0 \;\; \rightarrow \left\{ \begin{array}{l} \partial^a \partial_a X^\mu = 0 \\ \partial_\sigma X^\mu \Big|_{\sigma=\sigma_i} = 0 \end{array} \right.$$

Conclusion. The modified-measure (closed) string with N point-like ("color") charges on it is equivalent to N chain-wise connected regular open string segments with Neumann boundary conditions.

Classical Mechanism of "Color" Confinement

Recall that the modified string action yields the following eq. $\partial_\sigma \mathcal{E} + i[A_1, \mathcal{E}] + j^0 = 0$ for the dynamical string tension ($T \equiv |\mathcal{E}| = \Phi(\varphi)/\sqrt{-\gamma}$), which in the gauge $A_1 = 0$ reads:

$$\partial_\sigma \mathcal{E} - \sum_i C_i \delta(\sigma - \sigma_i) = 0 \quad , \quad \mathcal{E} \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \frac{F_{01}}{\sqrt{\mathrm{Tr}(F_{01}F_{01})}}$$

Note. This eq. of motion w.r.t. A_0 is in fact Hamiltonian first-class constraint – non-Abelian $D = 2$ "Gauss law" as in ordinary YM.

Let us consider the case of *closed* modified string with positions of the "color" charges $0 < \sigma_1 < \ldots < \sigma_N \leq 2\pi$. Then, integrating the "Gauss law" constraint we obtain:

$$\sum_i C_i = 0 \quad , \quad \mathcal{E}_{i,i+1} = \mathcal{E}_{i-1,i} + C_i$$

where $\mathcal{E}_{i,i+1} = \mathcal{E}$ for $\sigma_i < \sigma < \sigma_{i+1}$.

Conclusion. The only (classically) admissable configuration of "color" point-like charges coupled to a modified-measure closed bosonic string is the one with *zero* total "color" charge. The dynamically generated string tension is constant along each open string segment connecting two neighboring "color" charges with jumps at the ends of the segments proportional to the magnitude of the "end-point" charges.

Superstrings with a Modified Measure

Modified-measure Green-Schwarz superstring action:

$$S = \int d^2\sigma\, \Phi(\varphi)[-\frac{1}{2}\gamma^{ab}\Pi^\mu_a \Pi_{b\,\mu} + \frac{\varepsilon^{ab}}{\sqrt{-\gamma}}(\Pi^\mu_a(\theta\sigma_\mu\partial_b\theta) + \frac{1}{2}F_{ab}(A))]$$

(for simplicity we take now $G_{\mu\nu} = \eta_{\mu\nu}$) and where:

$$\Phi(\varphi) \equiv \frac{1}{2}\varepsilon_{ij}\varepsilon^{ab}\partial_a\varphi^i\partial_b\varphi^j \quad , \quad \Pi^\mu_a \equiv \partial_a X^\mu + i\theta\sigma^\mu\partial_a\theta$$

Here $\theta \equiv (\theta^\alpha)$ $(\alpha = 1,\ldots,16)$ denotes 16-dimensional Majorana-Weyl spinor in the embedding $D=10$ space-time; $\sigma^\mu \equiv \left((\sigma^\mu)_{\alpha\beta}\right)$ indicate the upper diagonal 16×16 blocks of the 32×32 matrices $C^{-1}\Gamma^\mu$ where Γ^μ and C are the $D=10$ Dirac and charge-conjugation matrices, respectively.

Explicit invariance under space-time supersymmetry transformations:

$$\delta_\epsilon\theta = \epsilon \quad , \quad \delta_\epsilon X^\mu = -i(\epsilon\sigma^\mu\theta)$$

$$\delta_\epsilon A_a = i(\epsilon\sigma_\mu\theta)\left(\partial_a X^\mu + \frac{i}{3}\theta\sigma^\mu\partial_a\theta\right)$$

In particular, the algebra of supersymmetry transformations closes on A_a up to a gauge transformation:

$$\{\delta_{\epsilon_1}, \delta_{\epsilon_2}\}\, A_a = \partial_a(-\frac{2}{3}(\epsilon_1\sigma^\mu\theta)(\epsilon_2\sigma_\mu\theta)) \; .$$

Hamiltonian Treatment. The canonical Hamiltonian is again linear combination of constraints only (now first- and second-class mixture).

The first part of (first- and second-class) constraints is the same as in the ordinary Green-Schwarz superstring case upon replacing the constant *ad hoc* string tension by the dynamical tension $E = \Phi(\varphi)/\sqrt{-\gamma}$:

$$\pi_{\gamma^{ab}} = 0 \ , \ \mathcal{T}_- \equiv \frac{1}{4}(\frac{\mathcal{P}}{E} - EX')^2 = 0$$

$$\mathcal{T}_+ \equiv \frac{1}{4}[\frac{\mathcal{P}}{E} + E(X' - 2i\theta\sigma\theta')]^2 - i\theta'\mathcal{D} = 0$$

$$i\mathcal{D} \equiv \mathcal{P}_\theta - (\mathcal{P}_\mu - E\Pi_{1,\mu})\, i\theta\sigma^\mu = 0$$

The last one is Lorentz non-covariant mixture of first- and second class constraints.

Solving the above problem – a long history of various approaches to super-Poincare covariant treatment of Green-Schwarz superstrings (E.N., Pacheva and Solomon; Galperin and Sokatchev; Cederwal; Bandos, Sorokin, Pasti, Tonin, Volkov; Berkovits; ...).

The second part of (first-class) constraint involving E and φ^i is the same as in the modified-measure bosonic string case, *i.e.*, "Gauss law" constraint for E and implying φ^i to be pure-gauge degrees of freedom.

Bosonic Branes with Modified-Measure

Ordinary p-brane action in Polyakov-type formulation:

$$S = -\frac{T}{2}\int d^{p+1}\sigma\,\sqrt{-\gamma}\,[\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \Lambda(p-1)]$$

Eqs. of motion w.r.t. γ^{ab} :

$$T_{ab} \equiv \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) G_{\mu\nu} + \gamma_{ab}\frac{\Lambda}{2}(p-1) = 0$$

The latter when $p \neq 1$ imply:

$$\Lambda\gamma_{ab} = \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}$$

or, equivalently:

$$T_{ab} \equiv \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{p+1}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) G_{\mu\nu} = 0$$

Note. Using $\Lambda\gamma_{ab} = \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}$, the Polyakov-type brane action is on-shell equivalent to the Nambu-Goto-type brane action:

$$S = -T\Lambda^{-\frac{p-1}{2}}\int d^{p+1}\sigma\,\sqrt{-\det||\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}||}$$

Modified Brane Action:

$$S = - \int d^{p+1}\sigma \, \Phi(\varphi)[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) + \frac{1}{\sqrt{-\gamma}}\Omega(A)] + \int d^{p+1}\sigma \, \mathcal{L}(A)$$

where the modified measure density is:

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!}\varepsilon_{i_1 \ldots i_{p+1}}\varepsilon^{a_1 \ldots a_{p+1}}\partial_{a_1}\varphi^{i_1} \ldots \partial_{a_{p+1}}\varphi^{i_{p+1}}$$

$\Omega(A)$ indicates a topological density given in terms of some auxiliary gauge/matter fields A^I living on the world-volume, "topological" meaning that:

$$\frac{\partial \Omega}{\partial A^I} - \partial_a \left(\frac{\partial \Omega}{\partial \partial_a A^I}\right) = 0 \text{ identically}$$

$$i.e. \;\; \delta\Omega(A) = \partial_a \left(\frac{\partial \Omega}{\partial \partial_a A^I}\delta A^I\right)$$

$\mathcal{L}(A)$ describes possible coupling of the auxiliary fields A^I to external "currents" on the brane world-volume.

The requirement for $\Omega(A)$ to be a topological density is dictated by the requirement that the modified-measure brane action (in the absence of the last "gauge/matter" term) reproduces the ordinary p-brane eqs. of motion apart from the brane tension $T \equiv \Phi(\varphi)/\sqrt{-\gamma}$ being now an *additional dynamical degree of freedom.*

Examples of Topological Densities for the Auxiliary Fields:

$$\Omega = -\frac{\varepsilon^{a_1 \dots a_{p+1}}}{p+1} F_{a_1 \dots a_{p+1}}(A) \quad , \quad F_{a_1 \dots a_{p+1}}(A) = (p+1)\partial_{[a_1} A_{a_2 \dots a_{p+1}]}$$

where $A_{a_1 \dots a_p}$ is rank p antisymmetric tensor (Abelian) gauge field.

More generally, for $p+1 = rs$:

$$\Omega = \frac{1}{rs} \varepsilon^{a_{11} \dots a_{1r} \dots a_{s1} \dots a_{sr}} F_{a_{11} \dots a_{1r}} \dots F_{a_{s1} \dots a_{sr}}$$

We may also use *non-Abelian* auxiliary gauge fields as in the string case. For instance, when $p = 3$ we may take:

$$\Omega = \frac{1}{4} \varepsilon^{abcd} \mathrm{Tr} \left(F_{ab}(A) F_{cd}(A) \right)$$

or, more generally, for $p+1 = 2q$:

$$\Omega = \frac{1}{2q} \varepsilon^{a_1 b_1 \dots a_q b_q} \mathrm{Tr} \left(F_{a_1 b_1} \dots F_{a_q b_q} \right)$$

where $F_{ab}(A) = \partial_a A_b - \partial_b A_c + i[A_a, A_b]$.

Eqs. of motion w.r.t. φ^i :

$$\frac{1}{2}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu G_{\mu\nu} + \frac{1}{\sqrt{-\gamma}}\Omega(A) = M \equiv \text{const}$$

Eqs. of motion w.r.t. γ^{ab} (assuming that $\int d^{p+1}\sigma\, \mathcal{L}(A)$ does not depend on γ_{ab} – true *e.g.* if it describes coupling of the auxiliary (gauge) fields A to charged lower-dimensional branes) :

$$\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} + \frac{\gamma_{ab}}{\sqrt{-\gamma}}\Omega(A) = 0$$

Both eqs. above imply:

$$\Omega(A) = -\frac{2M}{p-1}\sqrt{-\gamma} \;\; , \;\; \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} = \gamma_{ab}\frac{2M}{p-1}$$

$$\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{1}{p+1}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu G_{\mu\nu} = 0$$

The last two eqs. reproduce two of the ordinary brane eqs. of motion in the standard Polyakov-type formulation.

Eqs. of motion w.r.t. auxiliary (gauge) fields A^I – these are the eqs. determining the dynamical brane tension $T \equiv \Phi(\varphi)/\sqrt{-\gamma}$:

$$\partial_a\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\right)\frac{\partial\Omega}{\partial\partial_a A^I} + j_I = 0$$

where $j_I \equiv \frac{\partial\mathcal{L}}{\partial A^I} - \partial_a\left(\frac{\partial\mathcal{L}}{\partial\partial_a A^I}\right)$ is the corresponding "current" coupled to A^I.

For example, take $\Omega(A) = -\frac{\varepsilon^{a_1 \ldots a_{p+1}}}{p+1} F_{a_1 \ldots a_{p+1}}(A)$ and:

$$\int d^{p+1}\sigma \, \mathcal{L}(A) = \int d^{p+1}\sigma \, A_{a_1 \ldots a_p} j^{a_1 \ldots a_p}$$

Here $j^{a_1 \ldots a_p}$ is a current of charged $(p-1)$-sub-branes $\mathcal{B}_i$ embedded via $\sigma^a = \sigma_i^a(\underline{u})$ with parameters $\underline{u} \equiv (u^\alpha)_{\alpha=0,\ldots,p-1}$:

$$j^{a_1 \ldots a_p} = \sum_i e_i \int_{\mathcal{B}_i} d^p u \, \frac{1}{p!} \varepsilon^{\alpha_1 \ldots \alpha_p} \frac{\partial \sigma_i^{a_1}}{\partial u^{\alpha_1}} \cdots \frac{\partial \sigma_i^{a_p}}{\partial u^{\alpha_p}} \delta^{(p+1)}(\underline{\sigma} - \underline{\sigma}_i(\underline{u}))$$

The eq. for the dynamical brane tension $T \equiv \Phi(\varphi)/\sqrt{-\gamma}$ becomes:

$$\partial_a \Big(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\Big) + \sum_i e_i \mathcal{N}_a^{(i)} = 0$$

where $\mathcal{N}_a^{(i)}$ is the normal vector w.r.t. world-hypersurface of the $(p-1)$-sub-brane $\mathcal{B}_i$:

$$\mathcal{N}_a^{(i)} \equiv \frac{1}{p!} \varepsilon_{a b_1 \ldots b_p} \int_{\mathcal{B}_i} d^p u \, \frac{1}{p!} \varepsilon^{\alpha_1 \ldots \alpha_p} \frac{\partial \sigma_i^{a_1}}{\partial u^{\alpha_1}} \cdots \frac{\partial \sigma_i^{a_p}}{\partial u^{\alpha_p}} \delta^{(p+1)}(\underline{\sigma} - \underline{\sigma}_i(\underline{u}))$$

The X^μ-eqs. of motion $\partial_a \left(\Phi(\varphi)\gamma^{ab}\partial_b X^\mu\right) = 0$ (taking for simplicity $G_{\mu\nu} = \eta_{\mu\nu}$) become:

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} \partial_a \left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) - \sum_i e_i \mathcal{N}_a^{(i)} \gamma^{ab}\partial_b X^\mu = 0$$

which implies (∂_N – normal derivative w.r.t. $(p-1)$-sub-brane $\mathcal{B}_i$):

$$\partial_a \left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) = 0 \ , \quad \partial_N X^\mu \Big|_{\mathcal{B}_i} = 0$$

BRANE "CONFINEMENT"

$$T \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \quad , \quad \partial_a \left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}} \right) + \sum_{i=1}^{N} e_i \mathcal{N}_a^{(i)} = 0 \quad (*)$$

$\frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ ← Variable p-brane tension

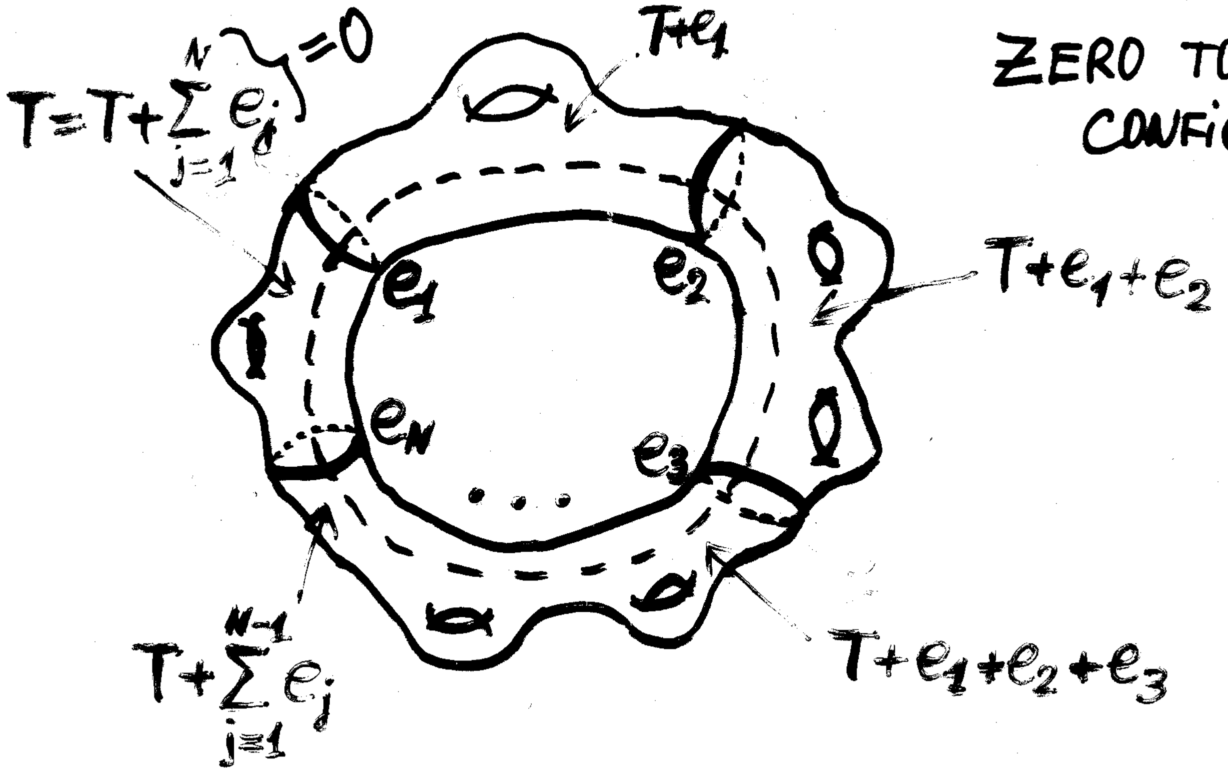
$\mathcal{N}_a^{(i)}$ ← normal w.r.t. (p-1)-sub-brane $\mathcal{B}_i$

($\mathcal{B}_i$'s do not intersect each other)

Integrate (*) along closed curve C on the p-brane world-volume:

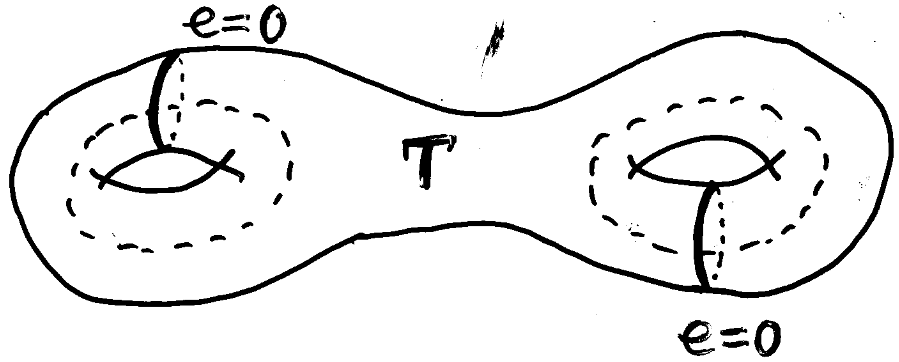
$$\oint_C (\ldots) = 0 \quad \Rightarrow \quad \sum_{i=1}^{N} n_i e_i = 0$$

$$n_i = \begin{cases} 0 & \text{- even number of } C \text{ crossing } \mathcal{B}_i \\ \pm 1 & \text{- odd number of } C \text{ crossing } \mathcal{B}_i \end{cases}$$

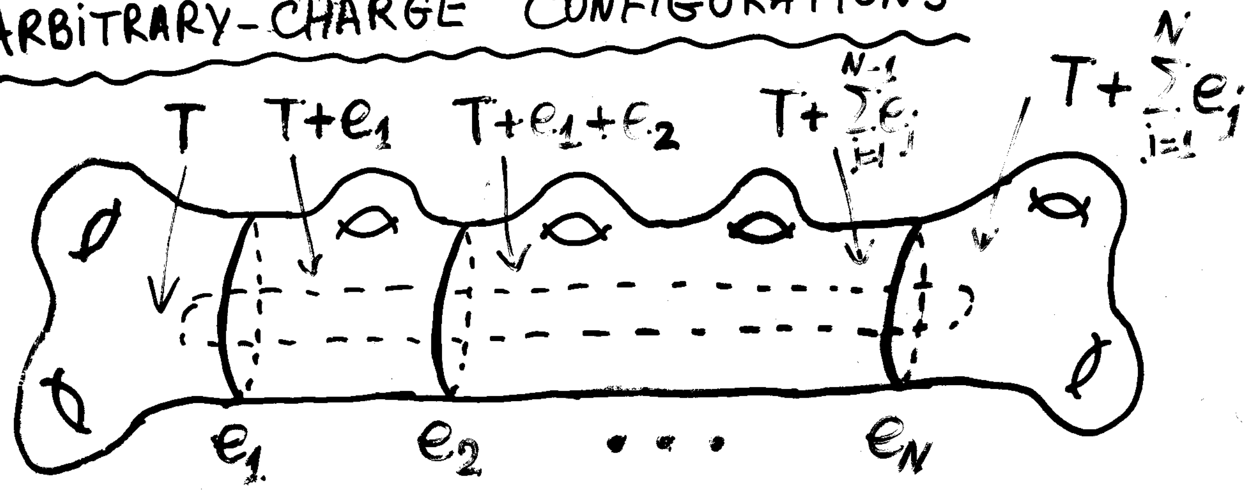


ZERO TOTAL CHARGE CONFIGURATION.

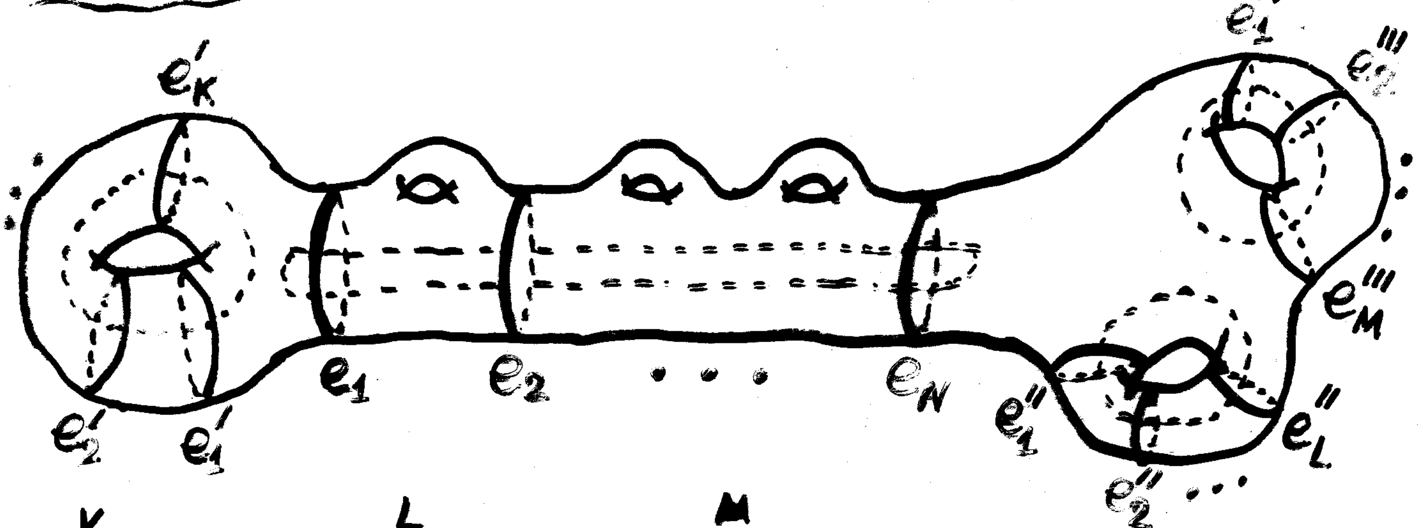
FORBIDDEN CONFIGURATIONS



ARBITRARY-CHARGE CONFIGURATIONS



MORE GENERAL CONFIGURATIONS



$$\sum_{j=1}^{K} e_j' = 0 \;,\; \sum_{j=1}^{L} e_j'' = 0 \;,\; \sum_{j=1}^{M} e_j''' = 0 \;,\; \{e_1, \ldots, e_N\}\text{-arbitrary}$$

Conclusions

Modifying of world-sheet (world-volume) integration measure – significantly affects string and brane dynamics.

- Acceptable dynamics *naturally* requires the introduction of auxiliary world-sheet gauge field (world-volume p-form tensor gauge field).

- String/brane tension - *not* a constant scale given *ad hoc*, but rather an *additional dynamical degree of freedom* beyond the ordinary string/brane degrees of freedom.

- The dynamical string/brane tension – physical meaning of an electric field strenght for the auxiliary gauge field.

- The dynamical string/brane tension obeys "Gauss law" constraint equation and may be nontrivially variable in the presence of point-like charges (on the string world-sheet) or charged lower-dimensional branes (on the p-brane world-volume).

- Modified-measure string/brane models provide simple classical mechanisms for confinement of point-like "color" charges or charged lower-dimensional branes due to variable dynamical tension.