MAXIMAL GAUGED SUGRAs IN THREE DIMENSIONS

H. Nicolai

AEI, Golm

based on:

H. Samtleben & H. N.:

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JHEP 0104 (2001) 022 [hep-th/0103032]

PL B514 (2001) 165 [hep-th/0106153]

T. Fischbacher: hep-th/0201030

T. Fischbacher, H. Samtleben & H. N.,
to appear

K. Koepsell, H. Samtleben & H. N.,
CQG 17 (2000) 3689

Previous results on gauged maximal SUGRA

$D = 4$ $\quad N = 8 \quad G_0 = SO(p, 8-p)$

de Wit, N. (1981) ; Hull (1984)

$D = 5$ $\quad N = 8 \quad G_0 = SO(p, 6-p)$

Pernici, Pilch, van Nieuwenhuizen (1985); Günaydin, Romans, Warner (1986)

$D = 7$ $\quad N = 4 \quad G_0 = SO(5)$

Pernici, Pilch, van Nieuwenhuizen (1984)

correspond to compactifications of $D = 11$ SUGRA

on $AdS_4 \times S^7 \quad (p = 0)$ — Freund, Rubin (1980); Duff, Pope (1981)

on $AdS_7 \times S^4$

and of $D = 10$ IIB SUGRA

on $AdS_5 \times S^5 \quad (p = 0) \longrightarrow$ AdS/CFT

Few results for $D = 3$:

- topological (CS) SUGRAs exist $\forall N$
  Achúcarro, Townsend (1986)
- dimensional reduction of $D \geq 4$ theories
  Deger, Kaya, Sezgin, Sundell;
- IIA on $AdS_3 \times S^7$ — Cvetič, Lu, Pope

but none of these maximally supersymmetric

Why is $D=3$ special ?

→ scalar – vector duality :

$$\varepsilon_{\mu\nu\rho}\,\partial^\rho \varphi^m = B_{\mu\nu}{}^m \equiv \partial_\mu B_\nu{}^m - \partial_\nu B_\mu{}^m$$

e.g. $D=11$ SUGRA on T^8 : (g_{MN}, A_{MNP})

g_{ij}	36	
A_{ijk}	56	
$g_{\mu i}$	8	$\Big\}$ dualize
$A_{\mu ij}$	28	

$$128 = \dim E_{8(8)}/SO(16)$$

↪ (ungauged) maximal $(N=16)$ SUGRA in three dimensions

Julia ; Marcus, Schwarz (1983)

[see deWit, N., Tollstén (1993) for a classification of $D=3$ theories ⇒ $N \leq 16$]

$$\boxed{\text{After dualization, no vector fields seem to be left for gauging!}}$$

Summary of results → JHEP 04 (2001) 022

- gauging involves both scalars <u>and</u> dual vectors → nonabelian duality
- Vector fields appear with CS term → <u>no</u> new propagating degrees of freedom (as required by SUSY!)
- More choices for gauge groups $\subset E_{8(8)}$

  $G_0 = E_8,\ E_7 \times A_1,\ E_6 \times A_2,$
  $F_4 \times G_2,\ D_4 \times D_4$

  (in various real forms)
- <u>Not</u> a compactification of $D = 11$ SUGRA by any known mechanism
- $E_{8(8)}$ broken, but still crucial
- Extremal structure of potential far richer than for $D \geq 4$

  → partial results.

  In particular, for $G_0 = SO(5,3) \times SO(5,3)$ potential has both AdS <u>and</u> dS stationary points, and

  $G_0 = SO(4,4) \times SO(4,4)$ has $\Lambda = 0$!

N = 16 SUGRA : a brief reminder

Physical (propagating) fields:

bosons $\varphi^A \in 128_s$ of $SO(16)$

fermions $\chi^{\dot{A}} \in 128_c$ of $SO(16)$

Non-propagating fields:

dreibein $e_\mu{}^\alpha$

gravitinos $\psi_\mu^I \in 16_v$ of $SO(16)$

Coset description of scalar fields:

$$\mathcal{V}(x) \in E_{8(8)} / SO(16)$$

with "hidden" global $E_{8(8)}$ and local $SO(16)$

$$\mathcal{V}(x) \longrightarrow g\, \mathcal{V}(x)\, h^{-1}(x)$$

unitary gauge: $\mathcal{V}(x) = \exp\left(\varphi^A(x) Y^A\right)$

For Lagrangian etc. we need

$$\mathcal{V}^{-1} \partial_\mu \mathcal{V} = \frac{1}{2} Q_\mu^{IJ} X^{IJ} + P_\mu{}^A Y^A$$

$$248 \longrightarrow 120 \oplus 128_s$$

$\rightarrow$ covariant derivatives, e.g.

$$\delta\psi_\mu{}^I = \hat{D}_\mu \varepsilon^I$$

$$= \left(\partial_\mu + \tfrac{1}{4}\omega_{\mu\alpha\beta}\gamma^{\alpha\beta}\right)\varepsilon^I + Q_\mu^{IJ}\varepsilon^J + \dots$$

Scalar field equation: $D^\mu P_\mu{}^A = \dots$

Using $E_{8(8)}$ indices $\mathcal{A} \equiv ([IJ], A)$, this can be rewritten as current conservation

$$\partial^\mu J_\mu{}^{\mathcal{M}} = 0$$

with $E_{8(8)}$ Noether current

$$J_\mu{}^{\mathcal{M}} = \mathcal{V}^{\mathcal{M}}{}_A P_\mu{}^A + \dots$$

Define 248 dual gauge fields via

$$J_\mu{}^{\mathcal{M}} = \varepsilon_\mu{}^{\nu\rho} B_{\nu\rho}{}^{\mathcal{M}}$$

modulo $U(1)^{248}$ gauge transformations

$$B_\mu{}^{\mathcal{M}} \longrightarrow B_\mu{}^{\mathcal{M}} + \partial_\mu \Lambda^{\mathcal{M}}$$

On-shell only!

Gauging: first steps

→ use a subset of the fields $B_\mu{}^{\mathcal{M}}$ to introduce nonabelian couplings

Possible gauge groups $G_0 \subset E_{8(8)}$ are characterized by embedding tensor

$$\Theta = \sum_i \varepsilon_i P_i = \Theta^T$$

where

P_i = projector onto i'th simple factor of G_0

ε_i = relative coupling strength

Shorthand: $B_\mu{}^{\mathcal{M}} \, \Theta_{\mathcal{M}\mathcal{N}} \, t^{\mathcal{N}} \equiv B_\mu{}^m \, t_m$

$$\mathcal{V}^{-1} \partial_\mu \mathcal{V} \rightarrow \mathcal{V}^{-1} (\partial_\mu + g \, B_\mu{}^m t_m) \mathcal{V}$$

$$= \frac{1}{2} Q_\mu{}^{IJ} X^{IJ} + \mathcal{P}_\mu{}^A Y^A$$

now g-dependent!

$$B_{\mu\nu}{}^m := \partial_\mu B_\nu{}^m - \partial_\nu B_\mu{}^m + g \, f^m{}_{np} B_\mu{}^n B_\nu{}^p$$

Lagrangian must be modified in order to preserve SUSY at $g \neq 0$.

idem for SUSY transformations.

New Lagrangian and consistency conditions

$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)}$$

with

$\mathcal{L}^{(0)} =$ as before, but with g-dependent Q_μ and P_μ

$$\mathcal{L}^{(1)} = -\frac{1}{4} g \, \varepsilon^{\mu\nu\rho} B_\mu{}^m \left(\partial_\nu B_{\rho m} + \frac{1}{3} g f_{mnp} B_\nu{}^n B_\rho{}^p \right)$$

$$\mathcal{L}^{(2)} = \frac{1}{2} g e \, A_1^{IJ} \, \bar{\psi}_\mu^I \gamma^{\mu\nu} \psi_\nu^J \; +$$

$$+ \, i g e \, A_2^{I\dot{A}} \, \bar{\chi}^{\dot{A}} \gamma^\mu \psi_\mu^I \; +$$

$$+ \, \frac{1}{2} g e \, A_3^{\dot{A}\dot{B}} \, \bar{\chi}^{\dot{A}} \chi^{\dot{B}}$$

$$\mathcal{L}^{(3)} = \frac{1}{8} g^2 e \left(A_1^{IJ} A_1^{IJ} - \frac{1}{2} A_2^{I\dot{A}} A_2^{I\dot{A}} \right)$$

$= -$ scalar potential

New gauge invariance:

$$\mathcal{V}(x) \longrightarrow g_0(x) \, \mathcal{V}(x) \, h^{-1}(x)$$

$$g_0(x) \in G_0 \, , \; h(x) \in SO(16)$$

SO(16) irreps for Yukawa couplings

A_1^{IJ} $\quad\quad \underset{\sim}{1} \oplus \underset{\sim}{135}$

killed by SUSY

$A_2^{I\dot{A}}$ $\quad\quad \underset{\sim}{1920} \oplus \cancel{\underset{\sim}{128}}$

$A_3^{\dot{A}\dot{B}}$ $\quad\quad \underset{\sim}{1} \oplus \underset{\sim}{1820} \oplus \cancel{\underset{\sim}{6435}}$

combine into irrep of $E_{8(8)}$!

$$\boxed{\underset{\sim}{135} \oplus \underset{\sim}{1820} \oplus \underset{\sim}{1920} = \underset{\sim}{3875}}$$

cf.

$D=5$ $\quad \underset{\sim}{36} \oplus \underset{\sim}{315}$ (of $USp(8)$) $= \underset{\sim}{351}$ (of $E_{6(6)}$)

$D=4$ $\quad \underset{\sim}{36} \oplus \underset{\sim}{420}$ + c.c. (of $SU(8)$) $= \underset{\sim}{912}$ (of $E_{7(7)}$)

For consistent gaugings, the SO(16) tensor A_1, A_2, A_3 can be constructed from the T-tensor

$$\boxed{T_{AB} := \mathcal{V}^{\mathcal{M}}{}_{A}\, \mathcal{V}^{\mathcal{N}}{}_{B}\, \Theta_{\mathcal{M}\mathcal{N}}}$$

Quadratic in $\mathcal{V}$ (rather than cubic) because adjoint (E_8) = fundamental (E_8)

Generally $T \in \left(\underset{\sim}{248} \otimes \underset{\sim}{248} \right)_{sym} =$

$$= \underset{\sim}{1} \oplus \underset{\sim}{3875} \oplus \underset{\sim}{27000}$$

Thus the consistency condition is

$$\boxed{\left(\mathbb{P}_{27000}\right)_{AB}{}^{CD}\, \Theta_{CD} \overset{!}{=} 0}$$

→ implies all other consistency requirements.

We have classified all solutions with at most two factors $G_0 = G_0^{(1)} \times G_0^{(2)}$

G_0	g_1/g_2
$SO(p,8-p) \times SO(p,8-p)$	-1
$G_{2(2)} \times F_{4(4)}$ $G_2 \times F_{4(-20)}$	$-3/2$
$E_{6(6)} \times SL(3)$ $E_{6(2)} \times SU(1,2)$ $E_{6(-14)} \times SU(3)$	-2
$E_{7(7)} \times SL(2)$ $E_{7(-5)} \times SU(2)$	-3
$E_{8(8)}$	

More specifically, $\Theta\big|_{27000} \stackrel{!}{=} 0 \Rightarrow$

$$\Theta_{IJ,KL} = -\frac{2}{7}\,\delta_{I[K}\,\Theta_{LJM,MJ]} + \Theta_{[IJ,KL]} + \frac{16}{7}\,\theta\,\delta^{IJ}_{KL}$$

$$\Theta_{IJ,A} = \frac{1}{7}\left(\Gamma_I\,\Gamma^K\right)_{AB}\,\Theta_{B,KJ}$$

$$\Theta_{A,B} = \frac{1}{96}\,\Gamma^{IJKL}_{AB}\,\Theta_{IJ,KL} + \theta\,\delta_{AB}$$

$$\Rightarrow \quad A_1^{IJ} = \frac{8}{7}\,\theta\,\delta^{IJ} + \frac{1}{7}\,T_{IK,KJ}$$

$$A_2^{I\dot{A}} = -\frac{1}{7}\,\Gamma^J_{A\dot{A}}\,T_{IJ,A}$$

$$A_3^{\dot{A}\dot{B}} = 2\theta\,\delta_{\dot{A}\dot{B}} + \frac{1}{48}\,\Gamma^{IJKL}_{\dot{A}\dot{B}}\,T_{IJ,KL}$$

For instance, $G_0 = SO(8)\times SO(8)$

$$\Theta_{IJ,KL} = \delta_{I[K}\,\Xi_{L]J}$$

$$\Theta_{IJ,A} = \Theta_{A,B} = 0$$

with $\Xi_{IJ} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \Rightarrow \Xi_{II} = 0$

Scalar Potential

$$\boxed{\mathcal{P}(\mathcal{V}) = \frac{1}{8} g^2 \left(\frac{1}{2} A_2^{I\dot{A}} A_2^{I\dot{A}} - A_1^{IJ} A_1^{IJ} \right)}$$

→ probably the most complicated potential ever studied!

Although a general study not feasible with present technology (even with computer algebra), the hidden $E_{8(8)}$ structure does help .....

Condition for stationarity:

$$\boxed{3 A_1^{IJ} A_2^{J\dot{A}} \overset{!}{=} A_3^{\dot{A}\dot{B}} A_2^{I\dot{B}}}$$

⇒ sufficient condition: $A_2^{I\dot{A}} = 0$

⇒ $\mathcal{V} = 1$ is a stationary point with maximal SUSY for all gauge groups.

(This is **not** true for $D = 4, 5$!)

To find stationary points $\mathcal{V} \neq 1$ restrict search to subspace of singlets for certain subgroups of G_0

(for $D = 4$: $SU(3) \subset SO(8)$ Warner (1984))

BACKGROUND ISOMETRIES OF THE MAXIMALLY SUPERSYMMETRIC GROUND STATES

Gauge Group	$N=(n_L, n_R)$	Supergroup $G_L \times G_R$
$SO(8) \times SO(8)$	$(8,8)$	$OSp(8\|2,\mathbb{R}) \times OSp(8\|2,\mathbb{R})$
$SO(7,1) \times SO(7,1)$	$(8,8)$	$F(4) \times F(4)$
$SO(6,2) \times SO(6,2)$	$(8,8)$	$SU(4\|1,1) \times SU(4\|1,1)$
$SO(5,3) \times SO(5,3)$	$(8,8)$	$OSp(4^*\|4) \times OSp(4^*\|4)$
$SO(4,4) \times SO(4,4)$	$(8,8)$	**Minkowski!**
$G_{2(2)} \times F_{4(4)}$	$(4,12)$	$D'(2,1;-\frac{2}{3}) \times OSp(4^*\|6)$
$G_2 \times F_{4(-20)}$	$(7,9)$	$G(3) \times OSp(9\|2,\mathbb{R})$
$E_{6(6)} \times SL(3,\mathbb{R})$	$(16,0)$	$OSp(4^*\|8) \times SU(1,1)^*$
$E_{6(2)} \times SU(2,1)$	$(12,4)$	$SU(6\|1,1) \times D'(2,1;-\frac{1}{2})$
$E_{6(-14)} \times SU(3)$	$(10,6)$	$OSp(10\|2,\mathbb{R}) \times SU(3\|1,1)$
$E_{7(7)} \times SL(2,\mathbb{R})$	$(16,0)$	$SU(8\|1,1) \times SU(1,1)^*$
$E_{7(-5)} \times SU(2)$	$(12,4)$	$OSp(12\|2,\mathbb{R}) \times D'(2,1;-\frac{1}{3})$
$E_{8(8)}$	$(16,0)$	$OSp(16\|2,\mathbb{R}) \times SU(1,1)^*$

* bosonic algebra

The Problem (and our approach)

- More than a dozen different gauge groups possible for $D = 3, N = 16$ SUGRA.

- Complexity of restricted potentials explodes when number of H-singlets increases.

- Head-on approach on a computer would typically involve intermediate quantities with $\mathcal{O}(10^6)$ subterms. Too much for many existing general-purpose symbolic algebra packages.

- Typical manipulations involve sparsely occupied higher-rank tensors; efficient support for this is rare in existing symbolic algebra packages. Suitable algorithms were developed for database applications in the 70's.

- Toolbox consisting of $\mathcal{O}(10^4)$ lines of compiled COMMON LISP code can do a typical 6-parameter potential in $\mathcal{O}(1 \text{ day})$ on a modern desktop machine.

→ T. Fischbacher

T. Fischbacher
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$$
\begin{aligned}
-8g^{-2}V \;=\;& \tfrac{189}{2} + \tfrac{1}{8}\,\mathrm{S}(3\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(3a)\mathrm{c}(4\varphi) \\
&+\tfrac{1}{8}\,\mathrm{S}(3\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(3a) - \tfrac{1}{8}\,\mathrm{K}(3\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{K}(\sigma_1)\mathrm{c}(4\varphi) \\
&-\tfrac{1}{2}\,\mathrm{S}(3\lambda_1)\mathrm{K}(2\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(3a)\mathrm{c}(4\varphi) + \tfrac{1}{2}\,\mathrm{S}(3\lambda_1)\mathrm{K}(2\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(3a) \\
&+\tfrac{3}{8}\,\mathrm{S}(3\lambda_1)\mathrm{S}(\sigma_1)\mathrm{c}(3a)\mathrm{c}(4\varphi) - \tfrac{5}{8}\,\mathrm{S}(3\lambda_1)\mathrm{S}(\sigma_1)\mathrm{c}(3a) \\
&-\tfrac{3}{8}\,\mathrm{S}(\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(3a)\mathrm{c}(4\varphi) - \tfrac{3}{8}\,\mathrm{S}(\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(3a) \\
&-\tfrac{3}{8}\,\mathrm{K}(\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{K}(\sigma_1)\mathrm{c}(4\varphi) + \tfrac{3}{2}\,\mathrm{S}(\lambda_1)\mathrm{K}(2\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(3a)\mathrm{c}(4\varphi) \\
&-\tfrac{3}{2}\,\mathrm{K}(3\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{K}(\sigma_1)\mathrm{c}(2a)\mathrm{c}(2\varphi) \\
&+\tfrac{3}{2}\,\mathrm{K}(\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{K}(\sigma_1)\mathrm{c}(2a)\mathrm{c}(2\varphi) \\
&+\tfrac{3}{2}\,\mathrm{S}(3\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(a)\mathrm{c}(2\varphi) \\
&+\tfrac{3}{2}\,\mathrm{S}(\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(a)\mathrm{c}(2\varphi) \\
&-\tfrac{3}{4}\,\mathrm{S}(\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(a) - \tfrac{3}{8}\,\mathrm{K}(3\lambda_1)\mathrm{K}(\sigma_1)\mathrm{c}(4\varphi) \\
&-\tfrac{3}{2}\,\mathrm{S}(\lambda_1)\mathrm{K}(2\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(3a) + \tfrac{3}{2}\,\mathrm{K}(3\lambda_1)\mathrm{K}(\sigma_1)\mathrm{c}(2a)\mathrm{c}(2\varphi) \\
&-\tfrac{3}{2}\,\mathrm{K}(\lambda_1)\mathrm{K}(\sigma_1)\mathrm{c}(2a)\mathrm{c}(2\varphi) - \tfrac{3}{2}\,\mathrm{S}(3\lambda_1)\mathrm{S}(\sigma_1)\mathrm{c}(a)\mathrm{c}(2\varphi) \\
&+\tfrac{3}{2}\,\mathrm{K}(\lambda_1)\mathrm{K}(2\lambda_2)\mathrm{K}(\sigma_1)\mathrm{c}(4\varphi) - \tfrac{9}{8}\,\mathrm{S}(\lambda_1)\mathrm{S}(\sigma_1)\mathrm{c}(3a)\mathrm{c}(4\varphi) \\
&+\tfrac{15}{8}\,\mathrm{S}(\lambda_1)\mathrm{S}(\sigma_1)\mathrm{c}(3a) + 12\,\mathrm{K}(2\lambda_1)\mathrm{K}(2\lambda_2)\mathrm{c}(2a)\mathrm{c}(2\varphi) \\
&-24\,\mathrm{S}(\lambda_1)\mathrm{K}(2\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(a)\mathrm{c}(2\varphi) + \tfrac{3}{2}\,\mathrm{c}(4\varphi) \\
&-12\,\mathrm{K}(2\lambda_1)\mathrm{c}(2a)\mathrm{c}(2\varphi) - 12\,\mathrm{K}(2\lambda_2)\mathrm{c}(2a)\mathrm{c}(2\varphi) \\
&-24\,\mathrm{S}(\lambda_1)\mathrm{K}(2\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(a) + 12\,\mathrm{c}(2a)\mathrm{c}(2\varphi) \\
&+\tfrac{9}{4}\,\mathrm{S}(3\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{S}(\sigma_1)\mathrm{c}(a) - \tfrac{9}{4}\,\mathrm{S}(3\lambda_1)\mathrm{S}(\sigma_1)\mathrm{c}(a) \\
&+\tfrac{45}{2}\,\mathrm{S}(\lambda_1)\mathrm{S}(\sigma_1)\mathrm{c}(a)\mathrm{c}(2\varphi) + \tfrac{99}{4}\,\mathrm{S}(\lambda_1)\mathrm{S}(\sigma_1)\mathrm{c}(a) \\
&-\tfrac{19}{8}\,\mathrm{K}(3\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{K}(\sigma_1) + \tfrac{1}{2}\,\mathrm{K}(3\lambda_1)\mathrm{K}(2\lambda_2)\mathrm{K}(\sigma_1)\mathrm{c}(4\varphi) \\
&-\tfrac{1}{2}\,\mathrm{K}(3\lambda_1)\mathrm{K}(2\lambda_2)\mathrm{K}(\sigma_1) + \tfrac{23}{8}\,\mathrm{K}(3\lambda_1)\mathrm{K}(\sigma_1) + 12\,\mathrm{K}(2\lambda_1) \\
&+36\,\mathrm{K}(2\lambda_1)\mathrm{K}(2\lambda_2) - \tfrac{9}{8}\,\mathrm{K}(\lambda_1)\mathrm{K}(4\lambda_2)\mathrm{K}(\sigma_1) \\
&-\tfrac{9}{8}\,\mathrm{K}(\lambda_1)\mathrm{K}(\sigma_1)\mathrm{c}(4\varphi) + \tfrac{93}{2}\,\mathrm{K}(\lambda_1)\mathrm{K}(2\lambda_2)\mathrm{K}(\sigma_1) \\
&+\tfrac{405}{8}\,\mathrm{K}(\lambda_1)\mathrm{K}(\sigma_1) + \tfrac{7}{2}\,\mathrm{K}(4\lambda_2) + \tfrac{1}{2}\,\mathrm{K}(4\lambda_2)\mathrm{c}(4\varphi) \\
&+14\,\mathrm{K}(2\lambda_2) - 2\,\mathrm{K}(2\lambda_2)\mathrm{c}(4\varphi)
\end{aligned}
$$

where

$$
\mathrm{c}(\alpha) = \cos(\alpha), \qquad \mathrm{K}(\sigma) = \cosh(\sigma), \qquad \mathrm{S}(\tau) = \sinh(\tau).
$$

Extremum	Location $(\sigma_1, \lambda_1, \lambda_2, a, \phi)$	Form of the scalar field	Cosmological constant $\Lambda = 4V$	Remaining group symmetry	Remaining super-symmetry
X_0	$(0,0,0,0,0)$	0	$-128\, g^2$	$SO(8)\times SO(8)$	$(n_L, n_R) = (8,8)$
X_1	$(-K_1, K_1, K_1, 0, 0)$	$\exp\left(K_1\ (G_1^+ + G_2^+ - S_1)\right)$	$-200\, g^2$	$SO(7)^+ \times SO(7)^+$	None
X_2	$(-K_2, K_2, -2\,K_1, \pi, \frac{\pi}{2})$	$\exp\left(-K_2\ (G_1^+ + S_1) - 2K_1\, G_2^-\right)$	$-416\, g^2$	$SU(4)$	None
X_3	$(-K_3, -K_3, -K_3, \pi, \frac{\pi}{2})$	$\exp\left(K_3\ (G_1^+ - S_1 - G_2^-)\right)$	$-288\, g^2$	$SU(3)\times SU(3)\times$ $\times U(1)\times U(1)$	$(n_L, n_R) = (2,2)$
X_4	$(K_6, K_7, K_9, \pi, \frac{\pi}{2})$	$\exp\left(-K_7\, G_1^+ + K_9\, G_2^- + K_6\, S_1\right)$	$K_{10}\, g^2$	$SU(3)\times U(1)\times U(1)$	None
X_5	$(-2\,K_2, 0, -2\,K_1, \frac{\pi}{2}, \frac{\pi}{2})$	$\exp\left(-2K_2\, S_1 - 2K_1\, G_2^-\right)$	$-416\, g^2$	$SU(4)^-$	None

where

$$
\begin{aligned}
K_1 &= \tfrac{1}{4}\ln\left(7+4\sqrt{3}\right) \approx 0.6584789 \\
K_3 &= \ln(1+\sqrt{2}) \approx 0.8813736 \\
K_5 &= 18 + 6\sqrt{33} + 6\,K_4 \approx 90.6357598 \\
K_7 &= \ln\left(\tfrac{1}{6}\sqrt{K_5}\right) \approx 0.4616649 \\
K_9 &= \tfrac{1}{2}\ln\left(\tfrac{7}{2} + \tfrac{1}{2}\sqrt{33} - \tfrac{1}{2}K_8\right) \approx -1.2694452 \\
K_{10} &= -28512\left(1453 + 253\sqrt{33} + 116\,K_8 + 20\sqrt{33}\,K_8\right)\left(3+\sqrt{33}+K_4\right)^{-2}\left(7+\sqrt{33}+K_8\right)^{-2} \\
&\quad \left(24 + 6\sqrt{33} - 3\,K_4 - \sqrt{33}\,K_4\right)^{-1} \approx -398.5705673
\end{aligned}
$$

$$
\begin{aligned}
K_2 &= \tfrac{1}{2}\ln\left(\tfrac{5}{2}+\tfrac{1}{2}\sqrt{21}\right) \approx 0.7833996 \\
K_4 &= \sqrt{6+6\sqrt{33}} \approx 6.3613973 \\
K_6 &= \ln\left(\tfrac{1}{18}\sqrt{K_5}\left(19+\tfrac{5}{3}\sqrt{33}+\tfrac{5}{3}K_4+\tfrac{1}{144}K_5^2-\tfrac{1}{7776}K_5^3\right)\right) \\
&\approx -1.3849948 \\
K_8 &= \sqrt{78+14\sqrt{33}} \approx 12.5866547
\end{aligned}
$$

Fischbacher
Samtleben, N.

e.g. $G_0 = SO(5,3) \times SO(5,3)$

consider $SO(5) \times SO(5) \times SO(3) \subset G_0 \rightarrow$

128 scalars decompose as

$$128 \longrightarrow (5,5;1) + (5,1;3) + (1,5;3) + \underline{(1,1;1)} + (1,1;3) + (1,1;5) + (4,4;1) + (4,4;3)$$

$\longrightarrow$ only one singlet $(\equiv \phi)$

$$\mathcal{P}(\exp\phi) = -\frac{65}{8} - \frac{3}{8}\cosh(16\phi) + \frac{15}{2}\cosh(8\phi)$$

with stationary points at

$\phi = 0 \Rightarrow \mathcal{P} = -\frac{65}{8} < 0$ AdS

$\phi = \frac{1}{8}\operatorname{arcosh} 5 \Rightarrow \mathcal{P} = +11 > 0$ dS

<u>or</u>: $G_0 = SO(4,4) \times SO(4,4)$

$\mathcal{P}(\mathbb{1}) = 0 \Rightarrow$ Minkowski!

A D>3 Ancestor ?

Gauged D=3 supergravities apparently cannot be obtained from D=11 or D=10:

- Conventional Kaluza Klein gives Yang Mills, <u>not</u> Chern Simons

- There are no 8-(7-manifolds) $\mathcal{M}$ with $\mathrm{Isom}(\mathcal{M}) = G_0$

Hints of an exceptional geometry ?

→ CQG <u>17</u> (2000) 3689

$$\begin{pmatrix} e_\mu{}^\alpha & B_\mu{}^m e_m{}^a \\ 0 & e_m{}^a \end{pmatrix} \qquad \text{11-bein}$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$\begin{pmatrix} e_\mu{}^\alpha & B_\mu{}^{\mathcal{M}} \mathcal{V}_{\mathcal{M}}{}^{\mathcal{A}} \\ 0 & \mathcal{V}_{\mathcal{M}}{}^{\mathcal{A}} \end{pmatrix} \qquad \text{251-bein}$$

$$B_\mu{}^{\mathcal{M}} = (\underbrace{B_\mu{}^m , B_{\mu mn}}_{} , \ldots)$$

(8 + 28) = dim. maximal commuting nilpotent subalgebra of $E_{8(8)}$

Cremmer, Julia, Lu, Pope

$$\delta B_\mu{}^{\mathcal{M}} = \partial_\mu \xi^{\mathcal{M}} + \dots\dots$$

with $\xi^{\mathcal{M}} = (\xi^m , \xi_{mn} , \dots\dots)$

diffeomorphism in eight internal dimensions

antisymmetric tensor gauge transformations

suggests extension to more coordinates

$$y^m \longrightarrow y^{\mathcal{M}} = (y^m , y_{mn} , \dots\dots)$$

such that (→ central charges!)

$$\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}(y) = \frac{\partial y^{\mathcal{M}}}{\partial y'^{\mathcal{N}}} \mathcal{V}'^{\mathcal{N}}{}_{\mathcal{A}}(y')$$

with transition functions

$$\frac{\partial y^{\mathcal{M}}}{\partial y'^{\mathcal{N}}} \in E_{8(8)}$$

Gauge groups $G_0 \subset E_{8(8)}$ as isometry groups for suitable compactifications?

But might need further extensions

$$E_{8(8)}/SO(16) \longrightarrow E_{9(9)}/SO(16)^\infty \longrightarrow E_{10(10)}/H_{10}$$

? ? ? ?