Kopaonik, Sep 02

Strings and Intersecting brane worlds

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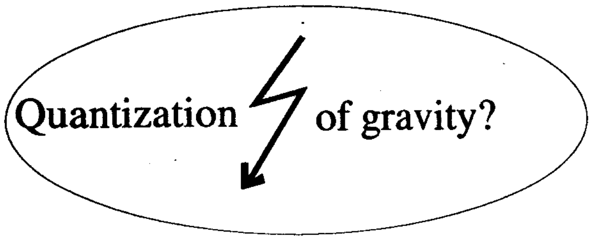
I) Introduction

2 columns of modern physics:

Quantum mechancs (QM):
Physical processes in the micro cosmos $\longrightarrow$
Standard model :
Gauge theory $G = SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$
plus 3 families of quarks und leptons.

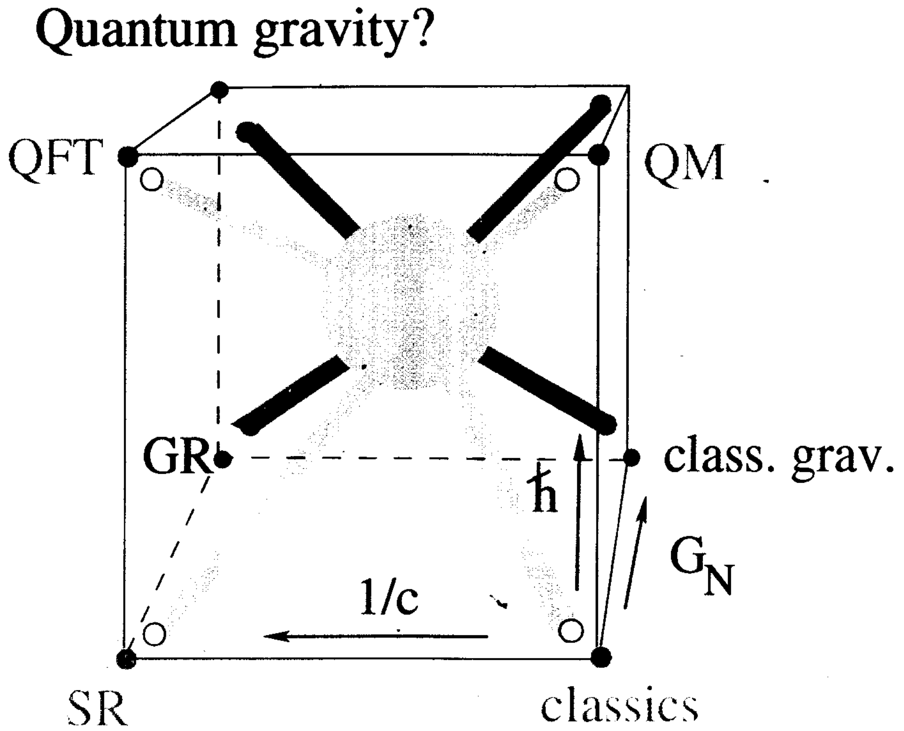


General relativity (GR):
gravity, physical processes in the macro cosmos $\longrightarrow$
Standard model of cosmology:
FRW model, inflationary universe

Search for **Quantum Gravity** as unification of both concepts?

Quantum gravity effects: physical processes in regions of large space-time curvature:
big bang, black holes
Structur of space and time at short distances?

The so far considered theories can be regarded as the vertices (edges) of a "magic cube":



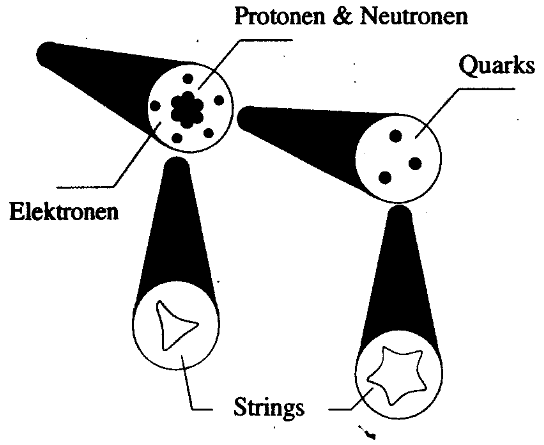
Key question: At which distances should quantum gravity effects become effective?

$$(G_N E^2 \simeq \alpha_{QED} \simeq O(1))$$

$$L_{\text{Planck}} = \sqrt{\frac{\hbar G_N}{c^3}} \simeq 10^{-33}\ \text{cm},\ M_{\text{Planck}} = \sqrt{\frac{\hbar c}{G_N}} \simeq 10^{19}\ \text{GeV}$$

Hierarchy problem: $M_{\text{Planck}} >> M_{\text{weak}} \simeq 100$ Gev ???

String theory:
Attempt to formulate a theory of quantum gravity



Plan of the talk:

II) (Closed heterotic) strings und their symmetries
(Unification of gravity and gauge interactions)

III) Brane world models ('Derivation' of the standard model)

IV) M-Theory on G_2 manifolds

I) Introduction

Some reasons why to think about string theory:

- Unification of particles and forces
- Questions in quantum gravity (astrophysics and cosmology – black holes, big bang, inflation)
- Better understanding of gauge theories (confinement, mass gap, ...)

Big questions in string theory:

- Groundstate of string theory $\longrightarrow$ contact to the Standard Model(s)
- Basic understanding of the underlying string symmetries (string field theory)
- Space-time supersymmetry breaking

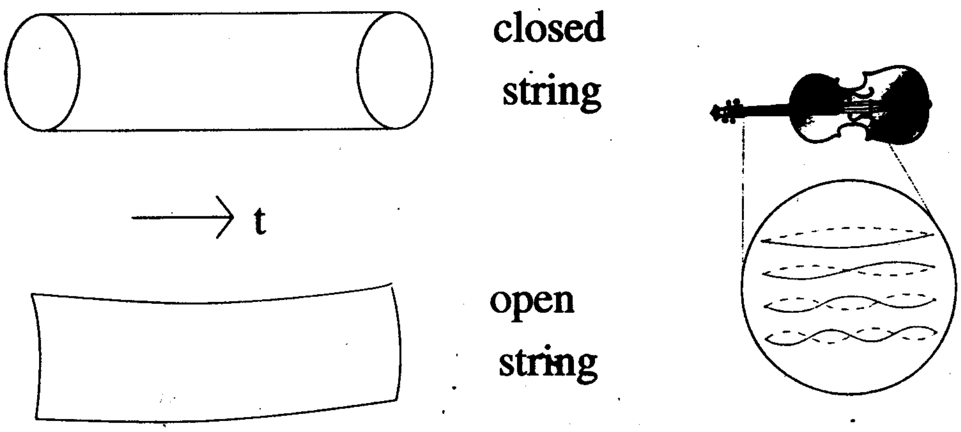
II) (Closed) strings and their symmetries

Super string theory:

(Veneziano (1968); Ramond (1971); Neveu, Schwarz (1971))

Elementary partices are not point like but rather 1-dim. objects

Main principle: there are only two elementary strings!



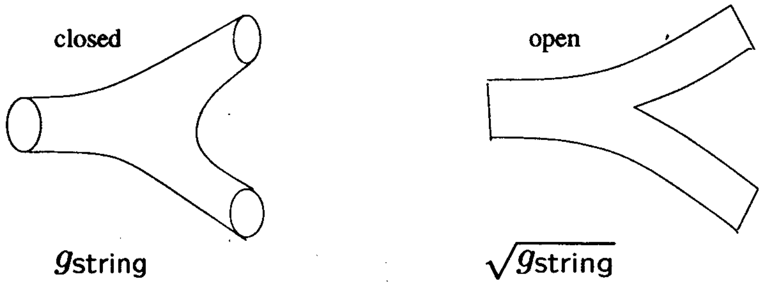
Elementary particles ⟷ Excitation modes of the 2 strings :

(i) Light excitations: $M \simeq 0$: quarks (u,d,c,s,t,b), leptons $(\nu_e, e, \nu_\mu, \mu, \nu_\tau, \tau)$, gauge bosons $(\gamma,\ W^\pm, Z^0, G)$

(ii) ∞-many heavy excitations: $M \simeq \sqrt{N} M_{\text{string}}$ $(N = 1,2,\dots)$: Exotic paricles with arbitrarily high spin

⟶ Unification of paricles and forces!

Interactions are due to the splitting and joining of strings:



Parameters of string theory:

(i) String coupling constant (dimensionless):

$$g_{\text{string}} = \langle S \rangle, \quad S = \text{dilaton field}$$

(ii) Mass of the string excitations (string tension τ):

$$M_{\text{string}} = \sqrt{\tau}\,, \quad M_{\text{string}} = \text{String mass}$$

(String length: $L_{\text{string}} = M_{\text{string}}^{-1} \quad (\hbar = c = 1)$)

Effective interactions of string modes $\Psi(x^\mu)$ (Electrons, quarks, gauge fields) $\longleftrightarrow$ Computation of the 2-dimensional string world sheet diagrams (2-dim. CFT):

$$S_{\text{eff}}[\Psi(x^\mu), g_{\text{string}}, M_{\text{string}}] \quad \longleftrightarrow \quad \text{Symmetrien?}$$

II) (closed) strings and their symmetries

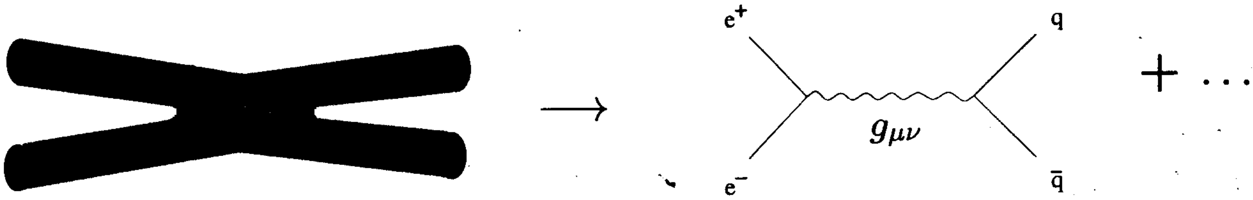
Purely closed (heterotic) string theory ($\leq$ 1995):

One particular, massless closed string mode: Spin 2 (graviton $g_{\mu\nu}$).

(Scherk, Schwarz (1974))

Quantum field theory of gravity:
Gravitational force $\longleftrightarrow$ Exchange of a massless closed string

GR $\longleftrightarrow$ effective theory



$$S_{\text{eff}}[g_{\mu\nu}] = \left(\frac{M_{\text{string}}^{D-2}}{g_{\text{string}}^{2}}\right) \int \mathrm{d}^D x \left(\sqrt{-g}\; R + \mathcal{O}(M_{\text{string}}^{-1})\right)$$

General coordinate covariance,	$M_{\text{Planck}}^{D-2} \simeq \frac{M_{\text{string}}^{D-2}}{g_{\text{string}}^{2}}$

Absence of ultraviolet divergencesn $\longrightarrow$
String theory: consistent realization of quantum gravity!
M_{string} is a natural cut-off.

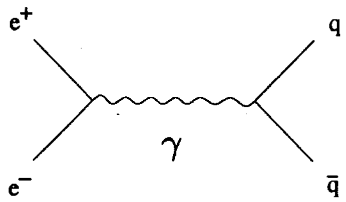
II) (Closed) strings and their symmetries

Heterotic string theory:

Gauge forces $\longleftrightarrow$ Exchange of closed strings

$\longrightarrow$ QED, QCD, weak interactions

$\longrightarrow$



$+ \dots$

$$S_{\text{eff}}[A\mu] = \left(\frac{M_{\text{string}}^{D-4}}{g_{\text{string}}^2}\right) \int \mathrm{d}^D x \left(F_{\mu\nu}^2 + \mathcal{O}(M_{\text{string}}^{-1})\right)$$

$$\boxed{\text{Gauge invariance,} \quad \frac{1}{g_{\text{gauge}}^2} \simeq \frac{M_{\text{string}}^{D-4}}{g_{\text{string}}^2}}$$

$\Longrightarrow$ Unification of gravity and gauge interactions!

$$\boxed{M_{\text{Planck}}^{D-2} \simeq \frac{M_{\text{string}}^2}{g_{\text{gauge}}^2} \longrightarrow M_{\text{string}} \simeq 10^{18}\ \text{GeV} \quad (D=4)}$$

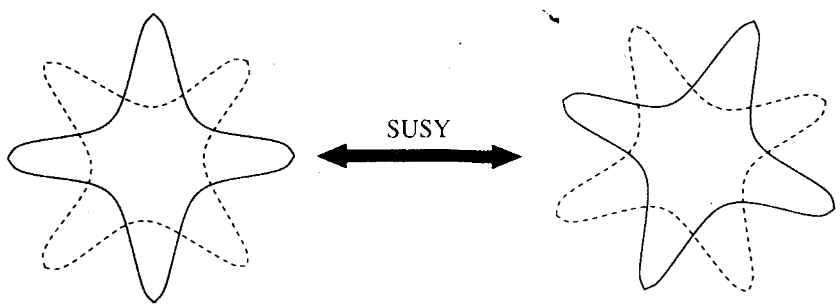
Further fundamental Symmetry: Supersymmetry

(Wess, Zumino (1974))

Fermions $\longleftrightarrow$ Bosons

$M_{\text{Susy}} \simeq \mathcal{O}(\text{TeV}) \rightarrow$ 'Solution' of the hierarchy problem
Realization of supersymmetry in string theory: there are always almost two identical closed string modes who are supersymmetric partners.

Fermion (quark) $\longleftrightarrow$ Boson (Squark)



Discovery of the supersymmetric partners at LHC, TESLA?

Theoretical problem: breaking of supersymmetry? (e.g.. by 'gaugino condensation')

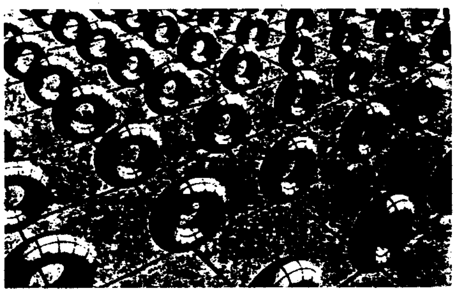
(Nilles; Font, Ibanez, D.L., Quevedo (1990))

Consistent formulation of string theory: Critical dimension!

A closed string can be only excited in 10 space-time dimensions.

Idea: Six space dimensions are curled up (compactified) to a very small internal space.
(Breaking of 10-dim. Lorentz symmetry!)

⟶ Compactification on 6-dimensional Calabi-Yau spaces
(Topology, algebraic geometry, mirror symmetry)



Question of the ground state of the theory: There are many ways of consistent compactifications⟶ the particle spactrum of string theory crucially depends on the choice of the internal space and hence is not unique.

Question: How can one detect an internal space of 'radius' R? – Send particles (strings) into the compact space (propagation in the compact space!)

Momenta in compact space directions are quantized:

$$p = \frac{n}{R}, \quad n = \pm 1, \pm 2, \ldots$$

Internal momentum states: massive repititions of the known particles (Kaluza-Klein particles) of mass

$$M_n = |n|/R$$

Experimental measurement of KK-particles:

(i) KK-particles of quarks, leptons, γ, G, $W^{\pm}$, Z^0: deviation from known gauge forces at short distances

$$R \leq (1 \text{ TeV})^{-1}$$

(ii) KK-particles of gravitons: deviation from the gravitational force at short distances

$$R \leq 1 \text{ mm}$$

How small can R be?

Besides the KK-particles, in string theory there exist also closed winding modes of mass

$$M_m = |m| \; M^2_{\text{string}} \; R, \quad m = \pm 1, \pm 2, \ldots$$

Invariance under T-duality symmetry (Exchange of ∞-many KK and winding modes):

$$R \longleftrightarrow L^2_{\text{string}}/R \quad \Longrightarrow \quad R \geq R_{\text{min}} \simeq L_{\text{string}}$$

Minimal measurable length scale that can be resolved by strings: : $\Delta X_{\text{min}} \simeq L_{\text{string}}$.

$$R = \langle T(x^\mu) \rangle \quad \Longrightarrow \quad S_{\text{eff}}[T(x^\mu)] = S_{\text{eff}}[1/T(x^\mu)]$$

T-duality invariance of $S_{\text{eff}} \longrightarrow$ automorphic funktionc:

E.g. radius dependent gauge coupling constant (1-loop):

$$\frac{1}{g^2_{\text{Eich}}} = \frac{1}{g^2_{\text{string}}} - \frac{b}{8\pi^2} \log \eta^2(T) + \ldots$$

(Ferrara, D.L. Shapere, Theisen (1989); Dixon, Kaplunovsky, Louis (1991))

Summary of closed string theory:

- Unification of gauge interactions (QM) and gravity (GR) $\longrightarrow$ Quantum gravity
- Critical dimension

Problems:

- Phenomenological interesting models?
- Experimental verification ($M_{\text{string}} \sim M_{\text{Planck}}$)?
- Ultimate, ∞-dimensional string symmetry, which acts on the ∞-many string degrees of freedom?

  (∞-dimensional Kac-Moody algebras?)

  (Damour, Henneaux, Julia, Nicolai (2001))

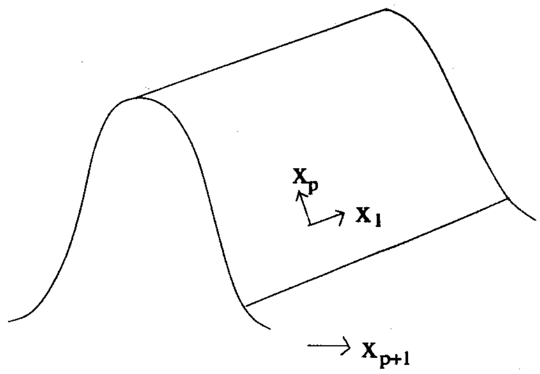
III) Brane-World-Models

S-duality: non-perturbative symmetry which relates the weakly couled regime of one theory with the strongly coupled regime of another/same theory (e.g. Ising model)

$$\text{Stringtheory}: \quad g_{\text{string}} \longleftrightarrow \frac{1}{g_{\text{string}}}$$

Exchange of elementary string excitations and solitonic string states ($M \sim g_{\text{string}}^{-1}$)!
(cfr. electrons $\leftrightarrow$ magnetic monopoles):

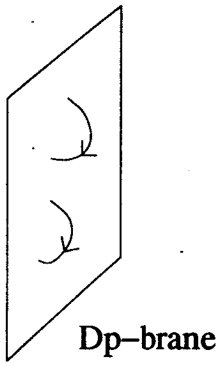
p-dimensional solitons: p-branes (supersymmetric solutions of the string equations of motion)



Question: Can one determine the string excitations (spectrum) around a p-brane solution?

Answer ($g_{\text{string}} \to 0$): Open strings which propagate along the world volume directions of the p-brane $\longrightarrow$ Dirichlet(p)-branes.

(Polchinski (1995))



D(p)-branes: (p)-dimensional topological defects, i.e. hyperplanes on which open strings are moving (tension: $\tau_p = M_{\text{string}}^{p+1}/g_{\text{string}}$).

Simplest D-brane configuration: 1 single Dp-brane:

Massless open string spectrum: $U(1)$ gauge boson $\longrightarrow$ supersymmetric $U(1)$ gauge theory in $p+1$ dimensions

Effective gauge interactions due to the exchange of open strings:

$$\begin{aligned} S_{\text{eff}} &= S_{DBI} = \tau_p \int \mathrm{d}^{p+1}x \sqrt{\det(g_{\mu\nu} + \tau^{-1}F_{\mu\nu})} \\ &= \left(\frac{M_{\text{string}}^{p-3}}{g_{\text{string}}}\right) \int \mathrm{d}^{p+1}x\, F_{\mu\nu}^2 + \dots \end{aligned}$$

The progress in non-perturbative string physics was made possible due the discovery of D-branes. *(Polchinski)*

D(p)-branes are higher(p)-dimensional topological defects, i.e. hypersurfaces, on which open strings are free to move.

They have led to several new insights:

- Effective (Non-Abelian) gauge theories on the world volumes of the D-branes → Brane world models

- Correspond to non-trivial gravitational backgrounds → Entropy count of black holes, AdS/CFT correspondence

- D-brane geometry:

  World volume directions: Non-commutative geometry

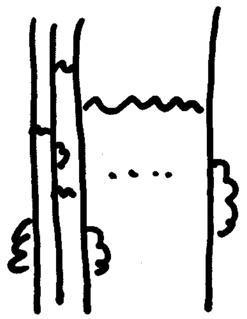
  $$[X_i, X_j] = i\Theta_{ij}$$

  Transversal directions: D-branes can serve as probes for the substringy regime

  $$L_{\text{D-brane}} = g_s L_{\text{string}} \leq L_{\text{string}}$$

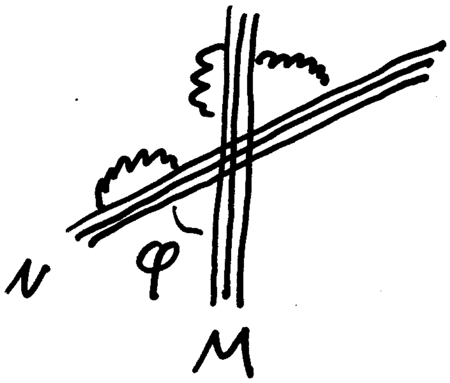
Other D-brane configurations

- N parallel Dp-branes



$\Rightarrow$ Supersymmetric $U(N)$ gauge theory in $p+1$ dimensions

- Intersecting D-branes



Open strings: (i) gauge bosons of $U(N) \times U(M)$

(ii) massless fermions in $(\underline{N}, \underline{M})$ repr.

(iii) massive scalars in $(\underline{N}, \underline{M})$ repr.

Important property of intersecting D-branes: they break space-time supersymmetry (unless certain angle conditions are satisfied)

This supersymmetry breaking manifests itself as the a massive/tachyonic scalar groundstate:

$$M_{ab}^2 = \frac{1}{2}\sum_I \Delta\varphi_{ab}^I - \max\{\Delta\varphi_{ab}^I\}$$

Massless scalars $\Leftrightarrow$ open string sector is supersymmetric.

In case the open string scalar is tachyonic ($M_{ab}^2 < 0$) $\longrightarrow$ the 2 different branes will recombine into a single brane:

Brane recombination $\longleftrightarrow$ Standard field theory Higgs effect

The type I string with Dp-branes ($\geq$ 1995):

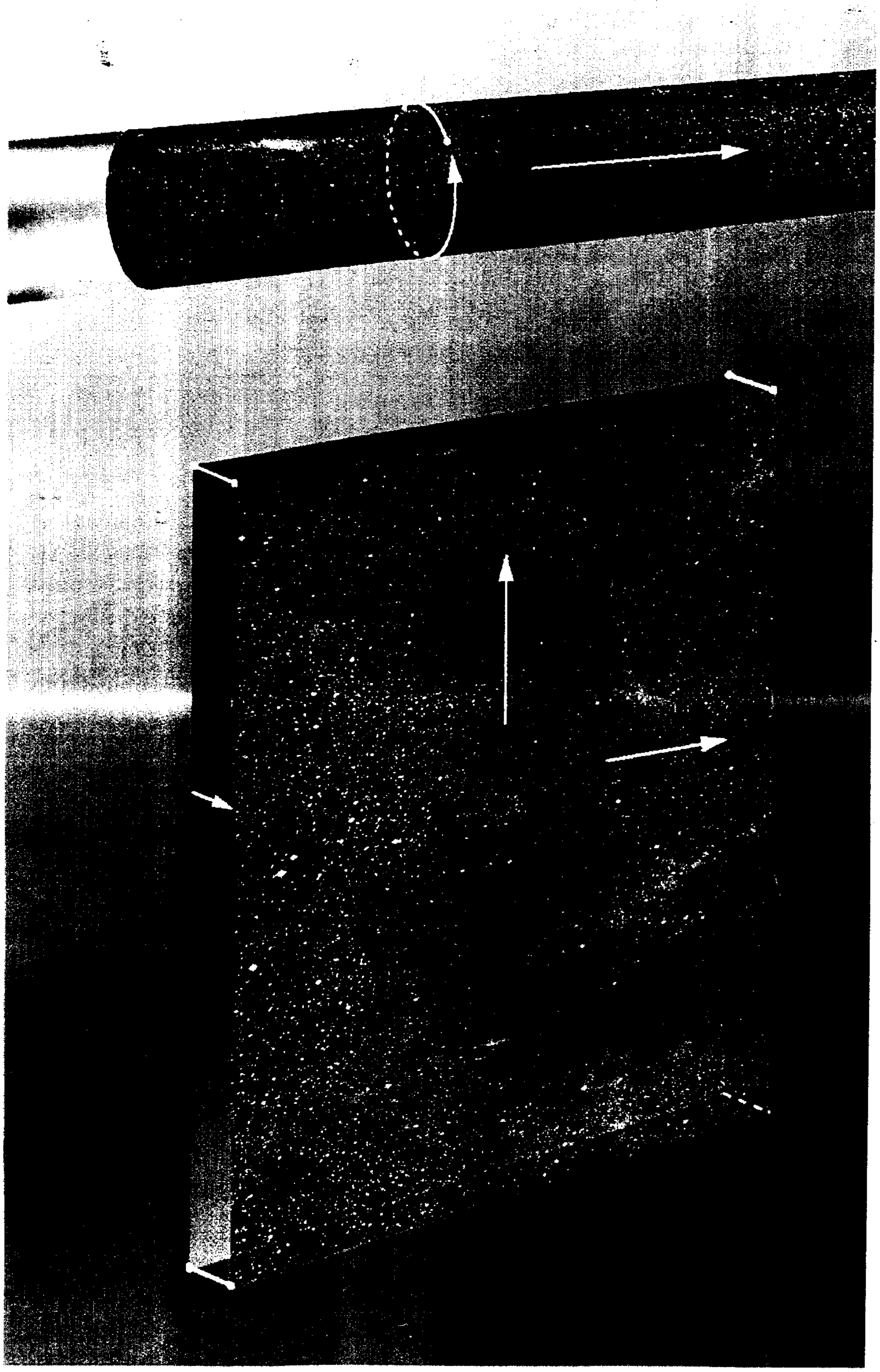
Type I string theory (orientifold of type II) contains closed and open strings

(i) Closed string: Graviton. I.e. the gravitational force is mediated by the exchange of closed strings which propagate in the 10-dimensional space-time (bulk space): $S_{\rm eff} \simeq (M^8_{\rm string}/g^2_{\rm string}) \int {\rm d}^{10}x\, R$

(ii) Open string: quarks, leptons, gauge bosons. I.e. the gauge forces are mediated by the exchange of open strings.

Open strings cannot propage in all 10 space-time directions but are confined to live on a p-dimensional hyper plane (boundary), namely the Dp-brane: $S_{\rm eff} \simeq (M^{p-3}_{\rm string}/g_{\rm string}) \int {\rm d}^{p+1}x\, F^2_{\mu\nu}$

$\longrightarrow$ "Universe" as a p-dimensional domain wall (Brane), which is embedded in the 10-dim. space-time!
(Dp-brane hat 3 uncompactified directions and $p-3$ internal, compact directions)



- Brane supersymmetry breaking: **In the open string spectrum supersymmetry is generically broken.**

- Possibility of large extra dimensions and a low string scale M_{string}:
  (Antoniadis, Arkani-Hamed, Dimopoulos, Dvali (1998))

$$g_{\text{Eich}}^{(4)} \simeq \sqrt{g_{\text{string}}}(M_{\text{string}} R_{\|})^{(3-p)/2}$$

$$M_{\text{Planck}}^{(4)} \simeq \frac{M_{\text{String}}}{g_{\text{string}}} \left(M_{\text{String}}^{3} \; R_{\|}^{(p-3)/2} R_{\perp}^{(9-p)/2} \right)$$

Hierarchy problem $\longrightarrow$ choose:

$M_{\text{string}}, R_{\|} \simeq \mathcal{O}(1 \text{ TeV}) \quad \Longrightarrow \quad R_{\perp} \simeq \mathcal{O}(1 \text{ mm}) \quad (p = 7)$

Exciting new experiments:

(i) Gravity: deviation from Newton law at short distances, high precision gravity experiments in the submillimeter region.

(ii) Discovery of the heavy string modes, i.e. resolution of the string length L_{string} in future accellerator experiments (Tevatron, LHC, TESLA)

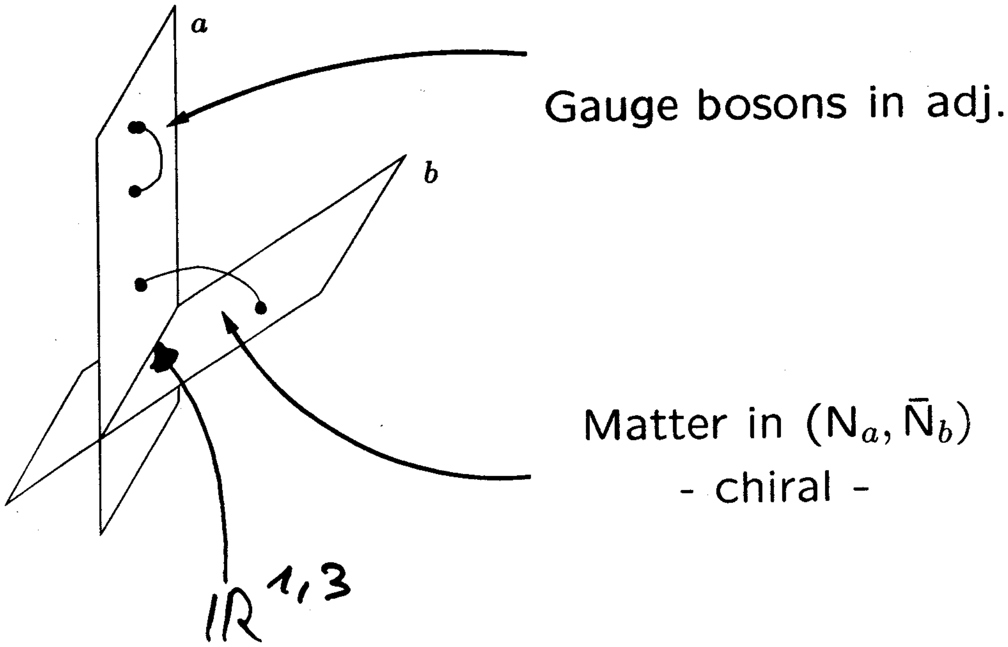
Concrete realization of the standard model:

Intersecting Brane-World-Models:

(Blumenhagen, Görlich, Körs, D.L. (2000/01/02))

Problem:
"Brane-engeneering" of the standard model gauge group,
Open strings as chiral quark- and lepton fields.

Solution:
Consider different D6-branes, which are wrapped around the homology 3-cycles of the internal Calabi-Yau space and which intersect each other in the internal space:
Number of chiral families correspond to the topological intersection numbers of the internal 3-cycles!



SM brane world model

- Three stacks of D6-branes with

$$N_1 = 3,\ N_2 = 2,\ N_3 = 1$$

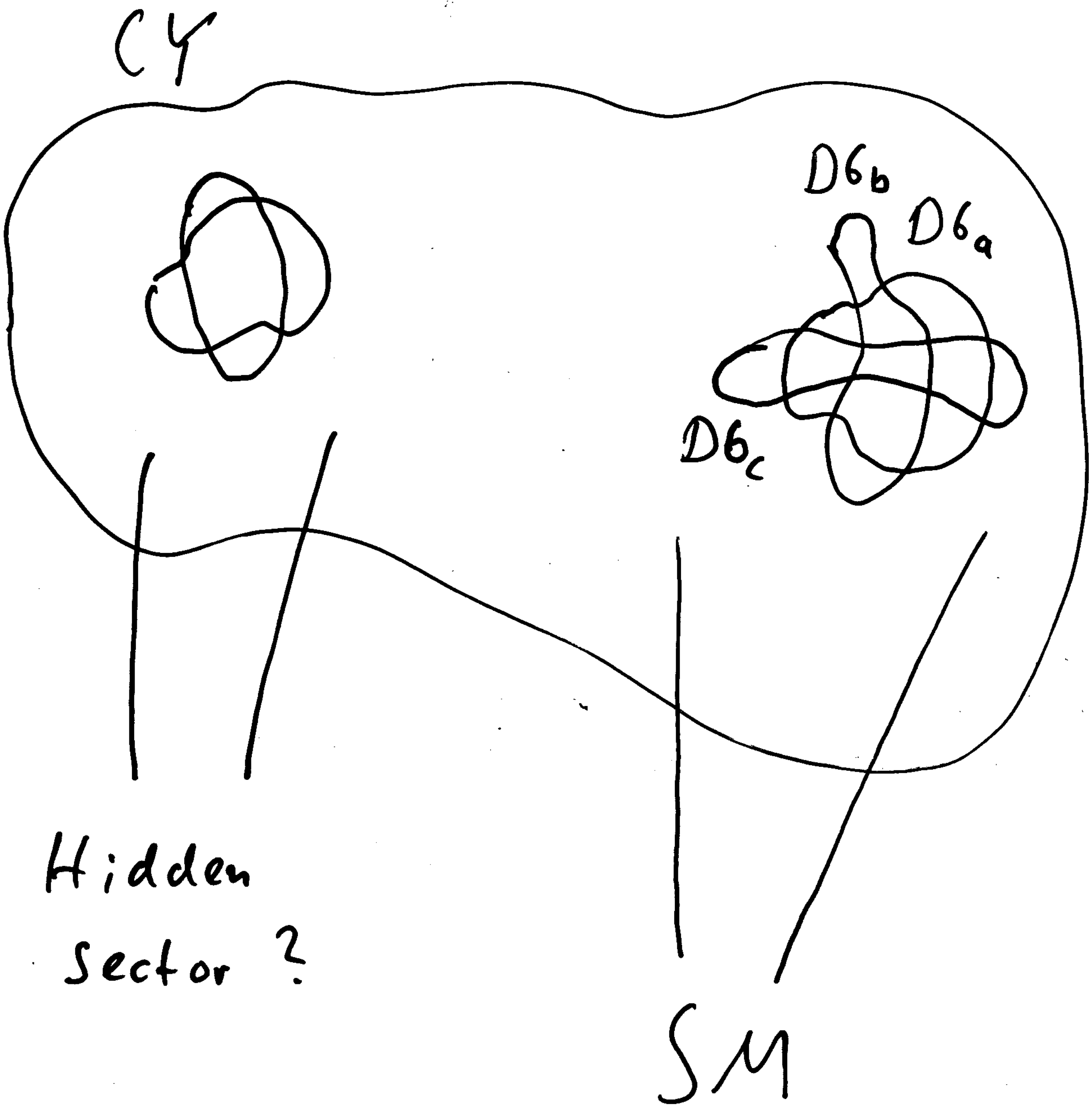
Gauge group : $G = SU(3)_c \times SU(2)_L \times U(1)_Y$

- SM spectrum from intersection numbers I_{ab}

Matter	Representation	$(Y, B-L)$
$(Q_L)_a$	$(\mathbf{3},\mathbf{2})$	$(1/3, 1/3)$
$(u^c_L)_a$	$(\bar{\mathbf{3}},\mathbf{1})$	$(-4/3, -1/3)$
$(d^c_L)_a$	$(\bar{\mathbf{3}},\mathbf{1})$	$(2/3, -1/3)$
$(l_L)_a$	$(\mathbf{1},\mathbf{2})$	$(-1, -1)$
$(e^+_L)_a$	$(\mathbf{1},\mathbf{1})$	$(2, 1)$
$(\nu^c_L)_a$	$(\mathbf{1},\mathbf{1})$	$(0, 1)$

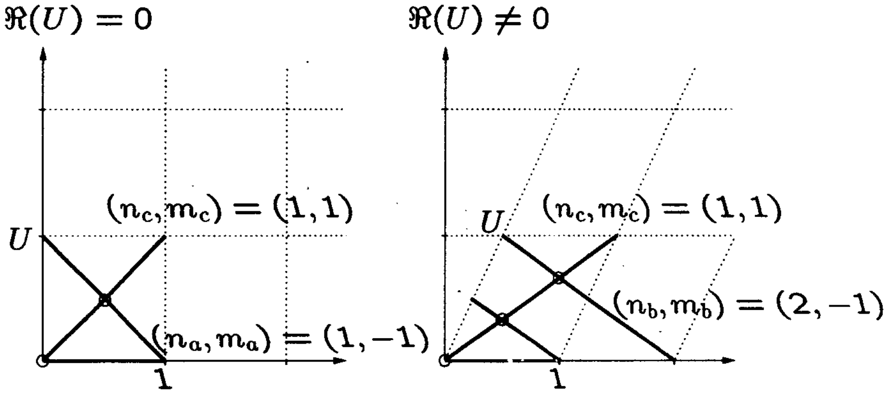
- The standard model Higgs field can be possibly identified with an open string tachyon $\rightarrow$ $SU(2)_L$ branes and $U(1)_Y$ brane annihilate into $U(1)_Q$ brane (Standard Higgs effect)

Intersecting branes on a CY space:



Models in 4d on tori

- Factorized complex tori $\mathbb{T}^6 = \mathbb{T}^2_1 \times \mathbb{T}^2_2 \times \mathbb{T}^2_3$

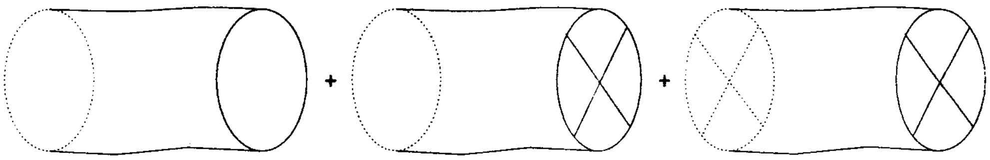


D6$_a$-branes: (n^I_a, m^I_a), O6-plane (in type I): $(1, 0)^3$

- Spectrum of chiral fermions: Intersection numbers

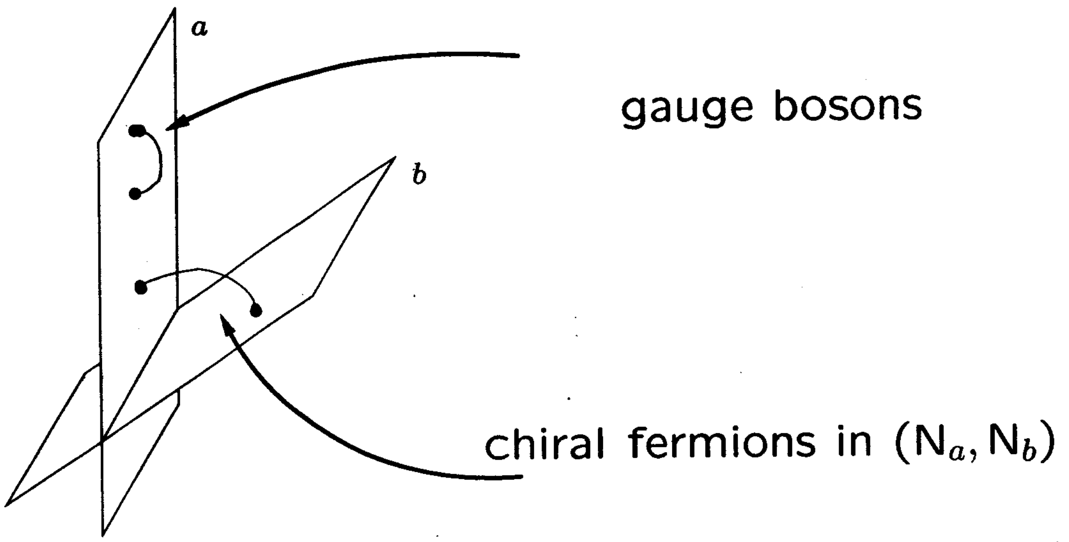
$$\chi \text{ in } (\mathrm{N}_a, \mathrm{N}_b): \quad I_{ab} = \prod_I \left(m^I_a n^I_b - m^I_a n^I_b \right)$$

- RR-charge cancellation - one-loop finiteness



$$\sum_a N_a \Pi[\mathrm{D6}_a] = 32\Pi[\mathrm{O6}] \text{ or } 0 \text{ or } 64\Pi[\mathrm{O6}]$$

Consider type IIA IBW's on 6-dim. CY spaces ($d = 3$):



- 7-dim. $U(N_a)$ gauge bosons on the D6-branes wrapped around sLag 3-cycle π_a $(codim = 3)$
- 4-dim. chiral fermions on the intersections of the D6-branes $(codim = 6)$:

Representation	Intersection #
$(\mathsf{A}_a)_L$	$\frac{1}{2}\left(\pi_a \circ \pi'_a + \pi_a \circ \pi_{O6}\right)$
$(\mathsf{S}_a)_L$	$\frac{1}{2}\left(\pi_a \circ \pi'_a - \pi_a \circ \pi_{O6}\right)$
$\left(\bar{\mathsf{N}}_a, \mathsf{N}_b\right)_L$	$\pi_a \circ \pi_b$
$(\mathsf{N}_a, \mathsf{N}_b)_L$	$\pi_a \circ \pi'_b$

Generalization of IBW's to K3 and Calabi-Yau spaces:

(i) $O(9-d)$ orientifold planes:
Choose the anti-holomorphic involution in local coordinates to be $\overline{\sigma} : z_i \to \overline{z}_i$. The orientifold plane $\pi_{\rm O}$, i.e. the fixed locus Fix($\Omega\overline{\sigma}$), is a sLag d-cycle, implying

$$\imath^* J = \imath^* \Im(\Omega_d) = 0, \quad \imath^* \Re(\Omega_d) = \imath^* d\text{vol}$$

(ii) Introduce N_a D(9-d) branes wrapped on sLag homology d-cycles π_a and their $\Omega\overline{\sigma}$ images π'_a.
For π_a only require $\imath^* \Re(e^{i\theta_a}\Omega_d) = \imath^* d\text{vol}$

Weak (individual) $\mathcal{N}=1$ SUSY conservation.
(Cremades, Ibáñez, Marchesano, hep-th/0201205)

Effective U(N) gauge theory on N D$(9-d)$-branes

$$\begin{aligned} \mathcal{S}_{\text{eff}} &= \int_{\mathbb{R}^4 \times \mathcal{M}^6} dx^4 d\xi^6 \, \mathcal{L}_{\text{gravity}}(g, B_2, \phi, C_{10-d}) \\ &+ \int_{\mathbb{R}^4 \times \mathcal{W}^{6-d}} dx^4 d\zeta^{6-d} \, (\underbrace{\mathcal{L}_{\text{DBI}}(g, \mathcal{F}, \phi)}_{\text{Tension}} + \underbrace{\mathcal{L}_{\text{CS}}(\mathcal{F}, C_{10-d})}_{\text{Charge}}) \end{aligned}$$

Example: **Fermat quintic CY_3:**

$$P(z_i) = z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0 \subset \mathbb{CP}^4$$

- O6-plane: sLag $\mathbb{RP}^3$, $\bar{\sigma}$-fixed set $\pi_{O6} = \pi_{0,0,0,0}$:

$$P(x_i) = x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 = 0 \subset \mathbb{RP}^4$$

- D6-branes: Use $\mathbb{Z}_5^4$, $z_i \mapsto \omega^{k_i} z_i$, $\omega = e^{2\pi i/5}$, $k_i \in \mathbb{Z}_5$ $\longrightarrow$ sLag 3-cycle $\pi_a = \pi_{k_2,k_3,k_4,k_5}$:

$$x_1^5 + \Re(\omega^{k_2} z_2)^5 + \Re(\omega^{k_3} z_3)^5 + \Re(\omega^{k_4} z_4)^5 + \Re(\omega^{k_5} z_5)^5 = 0$$

  $5^4 = 625$ sLag $\mathbb{RP}^3$'s, calibrated with $\Re(\prod_i \omega^{k_i} \Omega_3)$. (125 sLag's are $\Re(\Omega_3)$ calibrated $\longrightarrow \mathcal{N} = 1$ SUSY)

- Intersection numbers of π_{k_2,k_3,k_4,k_5} and π_{O6} from *Brunner, Douglas, Lawrence, Römelsberger, hep-th/9906200*

  (The 125 SUSY D6-branes have zero intersection #'s $\longrightarrow$ no $\mathcal{N} = 1$ chiral models from the quintic!)

Non-supersymmetric Standard Model from the quintic:

- Introduce four stacks of D6-branes with $N_a = 3$, $N_b = 2$ and $N_c = N_d = 1 \longrightarrow$ gauge group:

$$G = U(3) \times U(2) \times U(1)^2$$

- Choose the following "wrapping numbers":

$$\pi_a = |0,0,3,1\rangle, \quad \pi_b = |4,3,0,3\rangle,$$
$$\pi_c = |3,0,1,1\rangle - 2|4,3,0,3\rangle,$$
$$\pi_d = |4,2,4,4\rangle - 2|0,0,3,1\rangle$$

  This produces the intersection numbers of the standard model with

  3 generation of quarks and leptons

- Anomaly-free hypercharge is

$$U(1)_Y = \frac{1}{3}U(1)_a - U(1)_c + U(1)_d$$

- GS couplings to cancel $U(1) - SU(N)^2$ anomalies.

- An invisible sector needed for tadpole cancellation.

Further interesting aspects of IBW's

- Higgs effect by open string tachyon condensation and D-brane recombination.

- Mirror description of IBW's on the mirror Calabi-Yau space with D9-branes and magnetic gauge field fluxes.

- Problem of stabilty:

  (i) D-term potential due to FI-terms; this vanishes for supersymmetric D-brane configuations.

  (ii) F-term superpotential from disc and RP^2 instantons.

$$w(s, T^k) = \sum_{k,n,\vec{m}} \frac{1}{n^2} N_{k,\vec{m}} \exp(n[ks - \vec{m} \cdot \vec{T}])\,.$$

  These might destabilize the background. For certain simple non-compact CY threefolds using mirror symmetry such superpotentials have been explicitly computed.

(Aganagic, Vafa, hep-th/0012041)
(Aganagic, Klemm, Vafa, hep-th/0105045)
(Acharya, Aganagic, Hori, Vafa, hep-th/0202208)

Summary of brane world models:

- Phenomenological interesting models!

- Possible experimental consequences:

  Measurement of heavy string modes at colliders (LHC, Tesla)? ($M_{\text{string}} \leq \mathcal{O}(10 \text{ TeV})$)

  Measurement of extra dimensions? ($R_\perp \geq \mathcal{O}(1\ \mu\text{m})$)

Origin of chirality and Non-Abelian gauge groups in string theory:

- Heterotic string

- Type II superstring

- M-theory

Microscopic closed string consistency: world sheet modular invariane $\longrightarrow$ target space anomaly freedom

Microscopic open string consistency: cancellation of RR charges $\longrightarrow$ target space anomaly freedom

No microscopic consistency checks! But local anomaly cancellation by anomaly inflow on singularities

Different treatment of left- and rightmoving string modes
$\longrightarrow$ charged, chiral spectrum in closed string sector

D-branes (and orientifolds) $\longrightarrow$ charged, chiral spectrum in open string sector on D-branes

Put M-theory on a singular space (e.g. intervall $S^1/\mathbb{Z}_2$)
$\longrightarrow$ charged, chiral spectrum on end of the world branes

Compactification to 4D on an internal space $\mathcal{M}^6$ ($\mathcal{M}^7$):

Chirality $\longleftrightarrow$ Topology of $\mathcal{M}^6$ with vectorbundle V:

$$N_F = c_3(V)/2 \stackrel{V=T}{=} \chi(\mathcal{M}^6)/2$$

Chirality $\longleftrightarrow$ D-brane intersections on $\mathcal{M}^6$:

$$N_F = I_{ab} \equiv \#(\pi_a \cap \pi_b)$$

Chirality $\longleftrightarrow$ Singularities of $\mathcal{M}^7$, e.g. $\mathcal{M}^7 \simeq \mathbb{R}_+ \times Y_6$:

$$N_F = \chi(Y_6)/2$$

V) Summary

- Intersecting brane worlds with D6-branes on Calabi-Yau spaces provide an interesting framework in order to construct models with attractive phenomenological properties.
  The chiral spectrum is determined by the topological intersection numbers.

- G_2 manifolds provide an natural M-theory lift of $\mathcal{N} = 1$ supersymmetric IBW's.
  However it is very hard to construct explicit models with compact G_2 manifolds.