Super String Field Theory

and

Vacuum Super String Field Theory

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II SUMMER SCHOOL IN
MODERN MATHEMATICAL PHYSICS

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Kopaonik, (SERBIA) YUGOSLAVIA

- ***Why*** ***String Field Theory?***

main motivations:

gauge invariance principle behind string interactions

non-perturbative phenomena

- ***What*** ***we can calculate in String Field Theory?***

→ *Condensation of Tachyon* (analog of Higgs and *Goldstone Phenomena)*

Brane tensions

Rolling Tachyon

- ***How*** ***we can do calculations in SFT?***

String Field Theory and Noncommutative Geometry

String Field Theory and CFT

String Field Theory ≡
Second Quantized String Theory

- Infinite number of local fields

- String Field A[X(σ)] - functional, or state in Fock space

$$A = \sum_{\mathbf{n}} \int dp \, \Phi_{\mathbf{n}}(p) \left| \mathbf{n}, p \right\rangle$$

Local space-time fields

- Ghosts A[X(σ), c(σ), b (σ)]

- Example:

$$A = \int dp \left(t(p) c_1 + A_\mu(p) c_1 \alpha^\mu_{-1} + \lambda(p) c_0 \right) |p\rangle$$

INTERACTION ? $\left(A\left[x(\sigma),c(\sigma),b(\sigma)\right]\right)^3$?

Associative Product of String Fields

Examples of associative multiplications:

- ***Pointwise multiplication of functions*** $f(x)g(x)$
- ***Multiplication of matrices*** $\sum_j M_{ij}K_{jl}$
- ***Moyal product*** $(f*g)(x)=\exp\left[-i\theta^{\alpha\beta}\frac{\partial}{\partial x_\alpha}\frac{\partial}{\partial x'_\beta}\right]f(x)g(x')\Big|_{x=x'}$

Associative Product of String Fields -- Witten's String Product

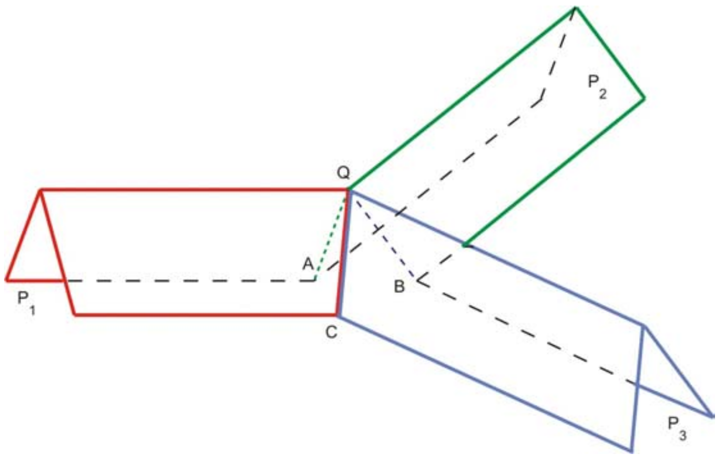
Coordinate representation

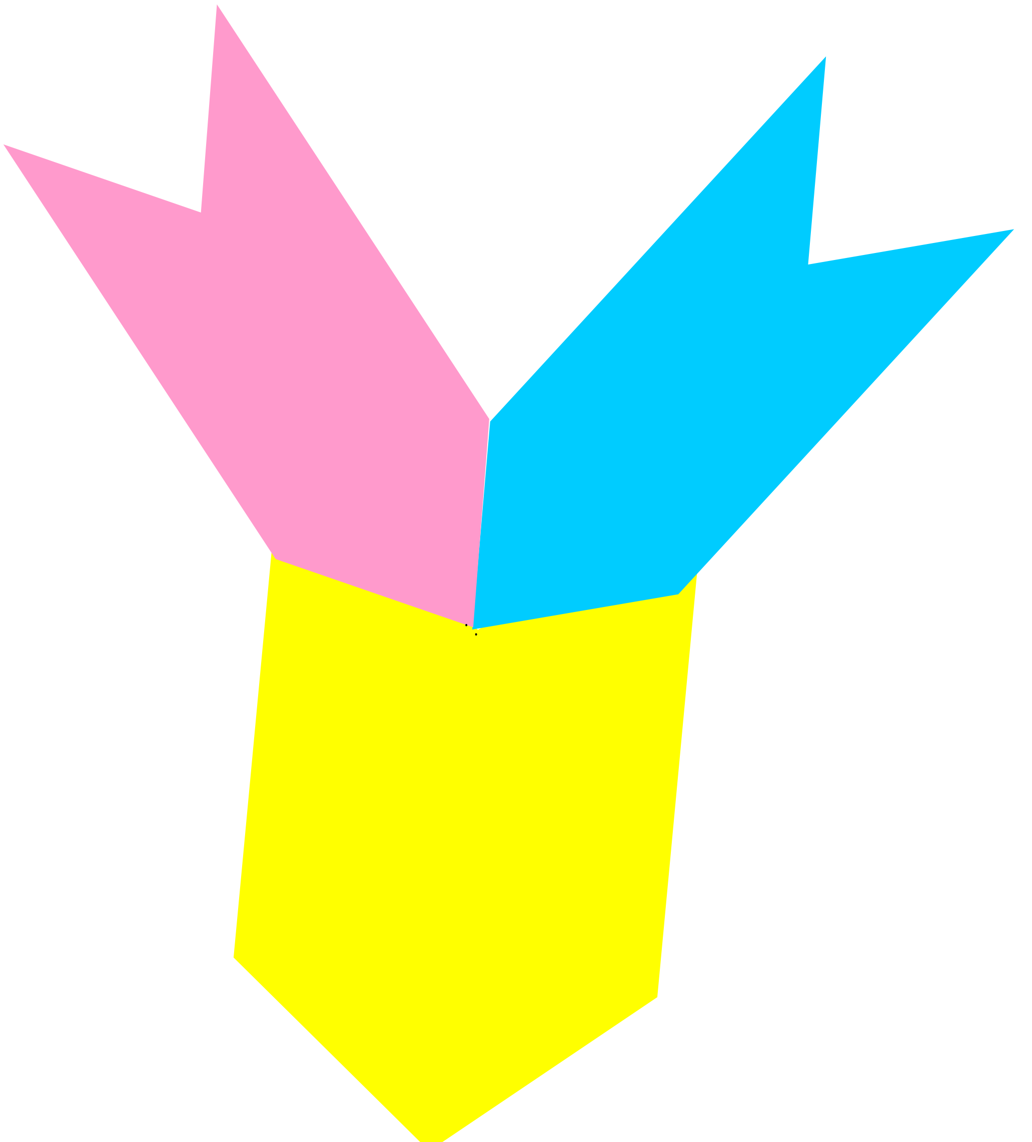
$$A[x(\sigma)] = \langle x(\sigma) | A \rangle$$

$$(A_1 * A_2)[z(\sigma)] \equiv \int A_1[x(\sigma)] A_2[y(\sigma)]$$

$$\times \prod_{\frac{\pi}{2} \le \sigma \le \pi} \delta[x(\sigma) - y(\pi - \sigma)] \prod_{\frac{\pi}{2} \le \sigma \le \pi} dx(\sigma) \prod_{0 \le \sigma \le \frac{\pi}{2}} dy(\sigma)$$

$$z(\sigma) = \begin{cases} x(\sigma) & \text{for } 0 \le \sigma \le \frac{\pi}{2} \\ y(\sigma) & \text{for } \frac{\pi}{2} \le \sigma \le \pi \end{cases}$$





SFT Action

E.Witten (1986)

$$S = -\frac{1}{g_o^2}\left(\frac{1}{2}\int A \star Q_B A + \frac{1}{3}\int A \star A \star A\right)$$

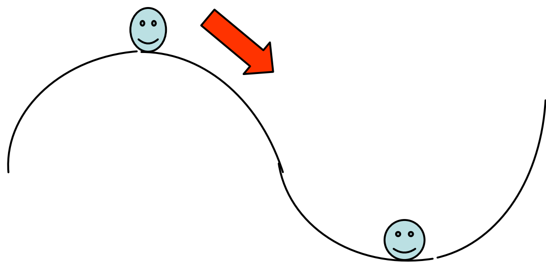
Gauge transformations: $\delta A = Q_B \Lambda + A \star \Lambda - \Lambda \star A$

- ***A*** - ***string field***
- Q_B - ***BRST charge, derivation of the star algebra***
- $\int$ - ***inner product***
- $\star$ - ***associative non-commutative product***
- g_0 - ***open string coupling constant***

SFT Action is given

Tachyon Condensation in SFT

- **Bosonic String - Tachyon**



$$A = \int dp \left(t(p)c_1 + A_\mu(p)c_1\alpha^\mu_{-1} + \lambda(p)c_0 \right) |p\rangle$$

$$S = -\frac{1}{g_o^2}\left(\frac{1}{2}\int A \star Q_B A + \frac{1}{3}\int A \star A \star A\right)$$

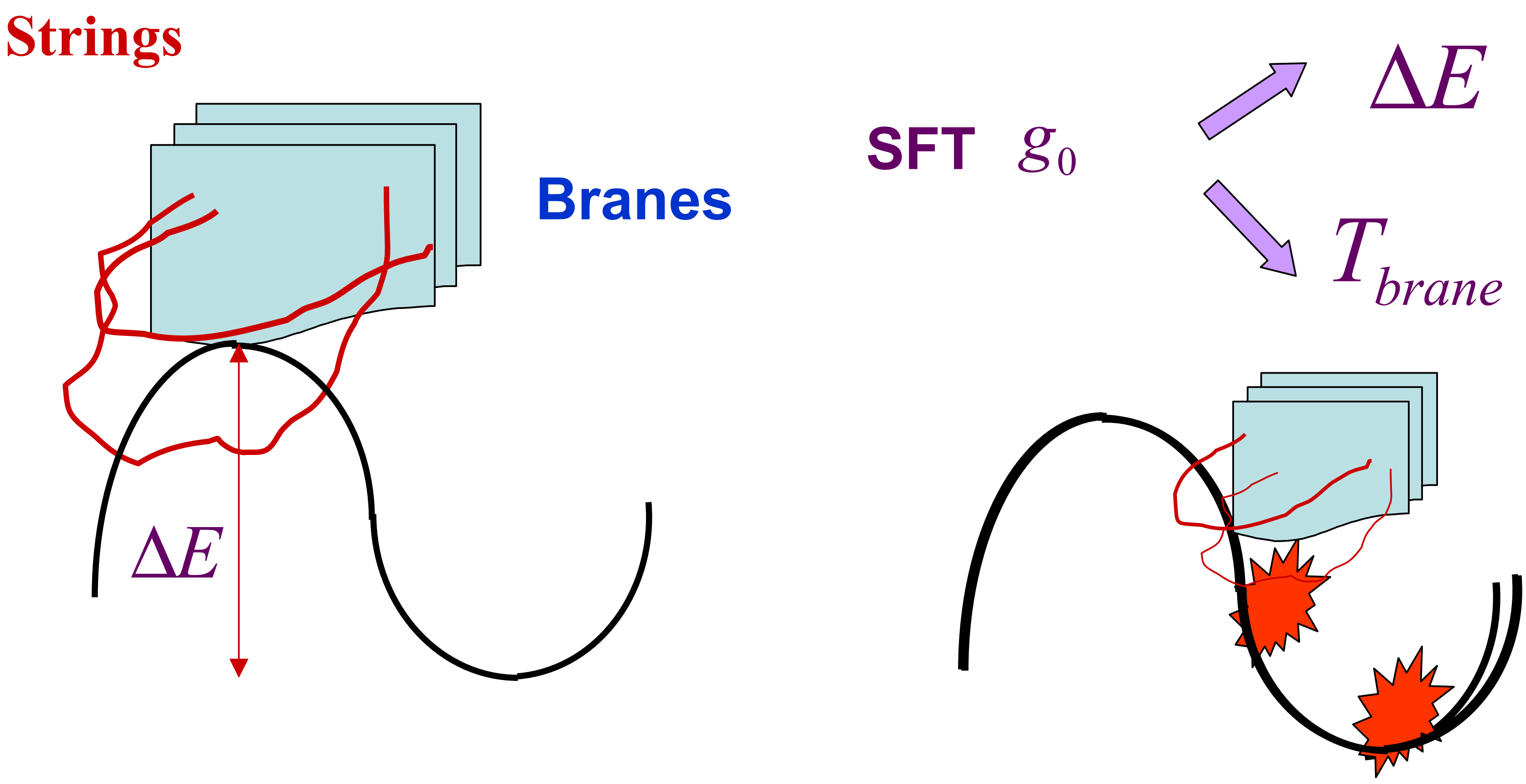
Kostelecky,Samuel (1989)

$$S = \frac{1}{g_0^2}\int dp\left[\frac{1}{2}t(-p)t(p)(-p^2+1)\right] + S_{\text{int}} \qquad \gamma = \frac{4}{3\sqrt{3}}$$

$$S_{\text{int}} = -\frac{1}{g_0^2}\int dx \frac{1}{3\gamma^3}\tilde{t}^3 \qquad \tilde{t} = ext\left[-\log\gamma\partial_\mu\partial^\mu\right] o t$$

Sen's conjecture (1999)

Vacuum Energy = Brane Tension



Sen's conjectures (1999)

- $$\Delta E = T_{brane}$$
- NO OPEN STRING EXCITATIONS
- CLOSED STRING EXCITATIONS

Rolling Tachyon

Motivations:cosmology,...

- Anharmonic oscillator
- alpha' corrections
- p-adic strings

I.Volovich(1987); P.Frampton, Nishino(1988); Brekke,Freund,Witten(1988), I.A,B.Dragovic,I.Volovich(1988)

Moeller, Zwiebach, hep-th/0207107

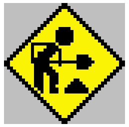
p-adic and NC space-time

I.A.,I.Volovich, 1990

- SFT

Sen, Strings2002

I.A, A.Giryavets, A.Koshelev



Rolling Tachyon

$$S = \frac{1}{g_0^2 \alpha'^{13}} \int dx \left(-\frac{\alpha'}{2} \partial_\mu t \partial^\mu t + \frac{1}{2} t^2 - \frac{1}{3\gamma^3} \tilde{t}^3 \right)$$

$$\tilde{t} = \exp\left[-\log \gamma \partial_\mu \partial^\mu \right] \circ t \qquad \gamma = \frac{4}{3\sqrt{3}}$$

Spatially homogeneous field configurations: $t(x) \equiv \varphi(x^0)$ $\quad x^0 \equiv t$

E.O.M.

$$-\frac{d^2}{dt^2}\varphi + \varphi - \frac{1}{\gamma^3} D_t (D_t \varphi)^2 = 0$$

where

$$D_t = \exp\left\{ \alpha' \ln \gamma \frac{d^2}{dt^2} \right\}$$

Rolling Tachyon

Anharmonic oscillator approximation

$$D_t = \exp\left\{ \alpha' \ln \gamma \frac{d^2}{dt^2} \right\} \approx 1$$

E.O.M.

$$-\frac{d^2}{dt^2}\varphi + \varphi - \frac{1}{\gamma^3}\varphi^2 = 0$$

Anharmonic oscillator

$$(\tfrac{d^2}{dt^2} + m_0^2)x(t) = j(t),$$

$$x(t) = x_0(t) + \int G_{ret}(t,t')\,j(t')\,dt'$$

$$x_0 = a\cos m_0 t$$

If $j = -\lambda x^3(t)$ **resonance** $x_0^3 = \tfrac{3}{4}a^3\cos m_0 t + \ldots$

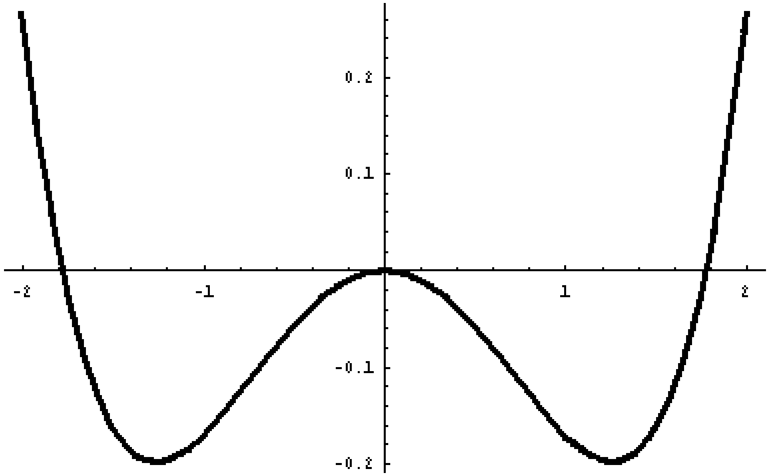
$$m_0 \Rightarrow m_0 + \frac{3\lambda a^2}{8m_0}$$

i.e. $x_0 = a\cos(m_0 + \frac{3\lambda a^2}{8m_0})t$

$$(\tfrac{d^2}{dt^2} - m_0^2)x(t) = -\lambda x^3(t)$$

$$x_0 = a\cosh(m_0 + \tfrac{3\lambda a^2}{8m_0})t$$

Rolling Tachyon

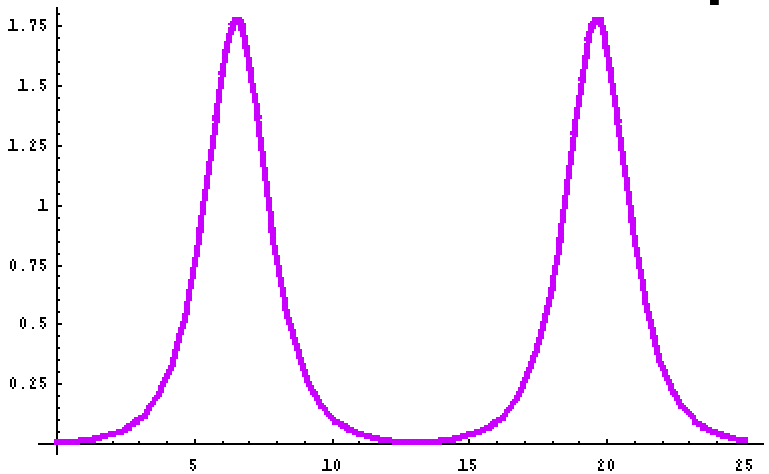


Two regimes:

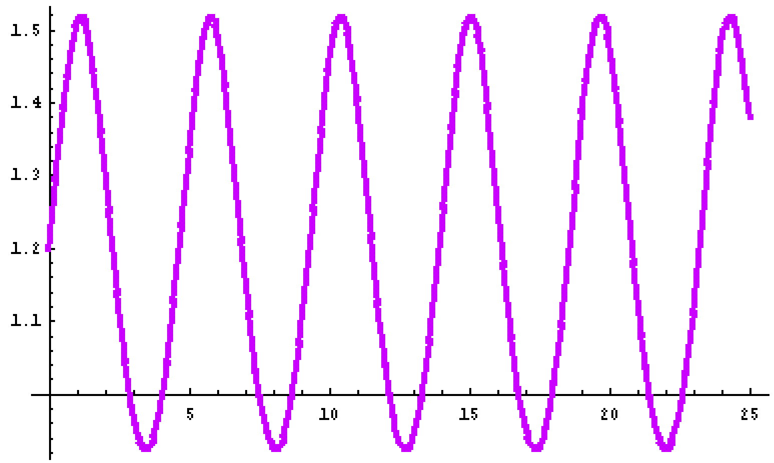
$$x_0 = a\cosh(m_0 + \tfrac{3\lambda a^2}{8m_0})t$$

$$x_0 = a\cos\sqrt{2}(m_0 + \tfrac{3\lambda a^2}{8m_0})t$$

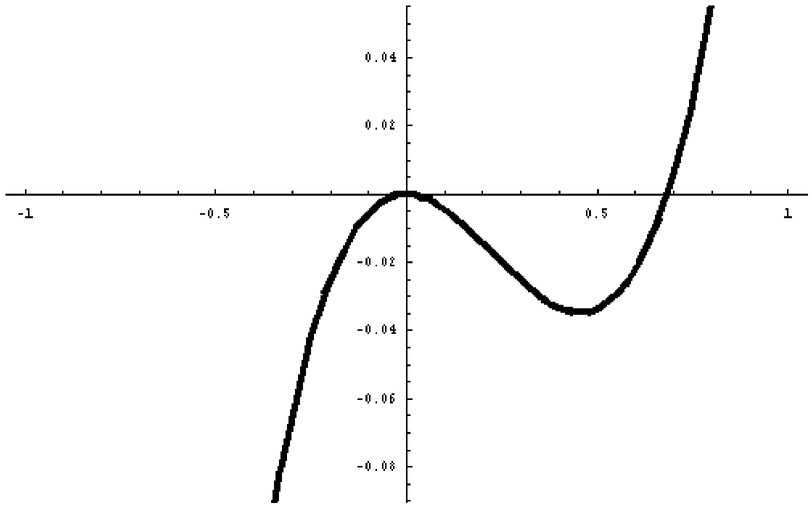
Initial condition near the top



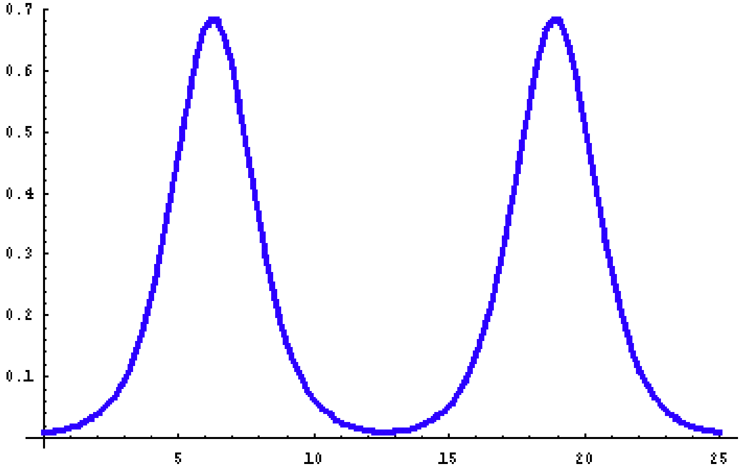
Initial condition near the bottom



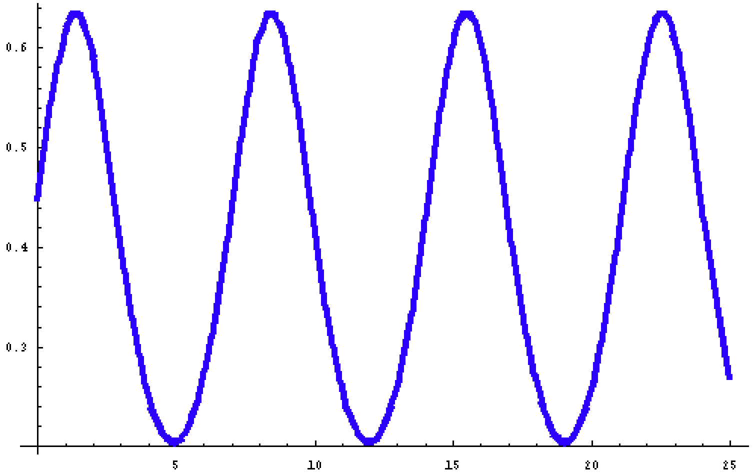
Rolling Tachyon (*bosonic case*)



Initial condition near the top



Initial condition near the bottom



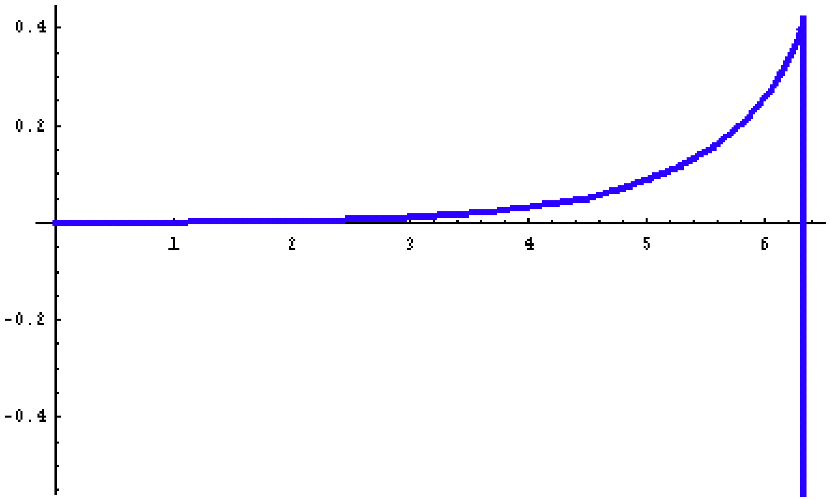
Alpha ' corrections (boson case)

- First order

$$\left(1+\frac{4\log\gamma}{\gamma^3}x\right)\frac{d^2}{dt^2}x(t)-x(t)+\frac{1}{\gamma^3}x^2(t)+\frac{2\log\gamma}{\gamma^3}\left(\frac{dx}{dt}\right)^2=0$$

Solutions

$$\gamma=\frac{4}{3\sqrt{3}}$$



Rolling Tachyon

E.O.M.

$$-\frac{d^2}{dt^2}\varphi + \varphi - \frac{1}{\gamma^3} D_t (D_t \varphi)^2 = 0$$

$$D_t \varphi(t) \equiv \exp\left\{\alpha' \ln \gamma \frac{d^2}{dt^2}\right\}\varphi(t) = \Im\, \varphi(t)$$

$$\Im_a \varphi(t) = \frac{1}{\sqrt{2\pi a}} \int e^{-\frac{(t-t')^2}{2a}} \varphi(t')\, dt'$$

$$\Im \varphi - \frac{1}{\gamma^3} \Im^2 (\Im \varphi)^2 = 0 \qquad (\Im \varphi)^2 \equiv \Phi$$

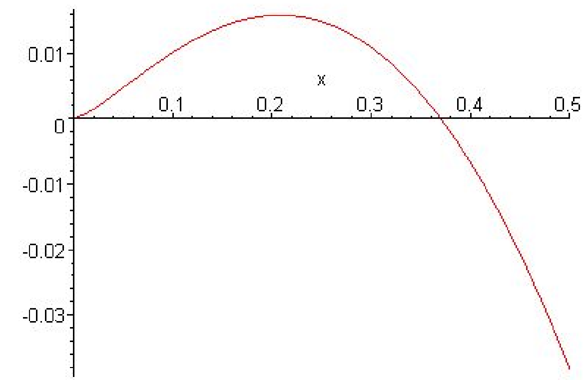
$$\Phi^{1/2} - \frac{1}{\gamma^3} \Im_{2a}(\Phi) = 0$$

$$a = 2\alpha' \ln \gamma$$

Rolling Tachyon

E.O.M.

$$\Phi^{1/2} - \frac{1}{\gamma^3}\Im_{2a}(\Phi) = 0$$



Two analogues:

i) p-adic strings

$$\Phi^{p} - \Im_{\ln p}(\Phi) = 0$$

ii) non-commutative solitons in strong coupling regime

Gopakumar, Minwalla,Strominger

$$\phi - \frac{1}{\gamma^3}\Im_{2a}(\phi^2) = 0 \quad \Longleftrightarrow \quad \phi - \phi * \phi = 0$$

Rolling Tachyon

Time evolution in the «sliver-tachyon» and p-adic strings

$$\Phi^p - \Im(\Phi) = 0$$

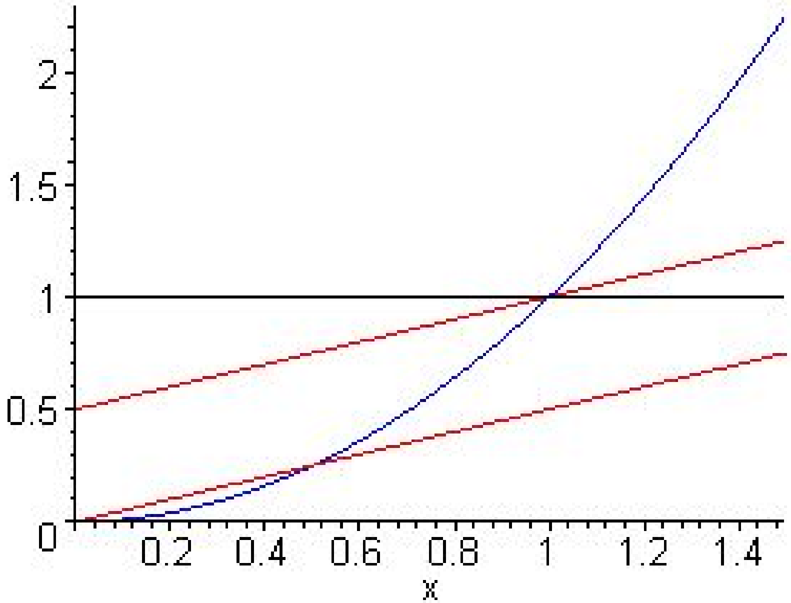
p=2,3,5,..

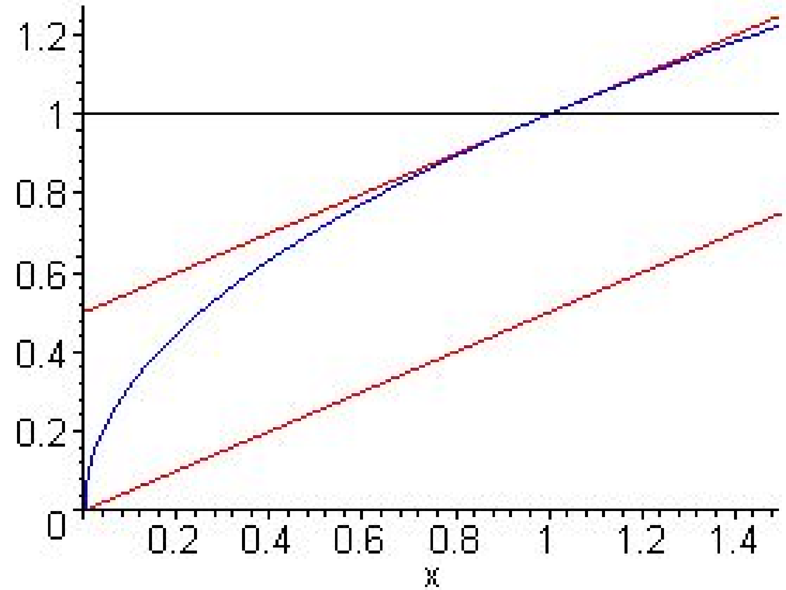
$$\Phi^{1/2} - \Im(\Phi) = 0$$

There is no solution such that

$\Phi(-\infty) = 1, \Phi(\infty) = 0$ **with** $\Phi(t)$

decreasing monotonically in time





Solutions to SFT E.O.M.

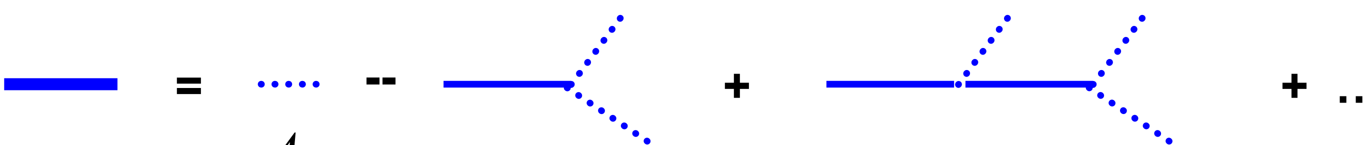
$$QA + A * A = 0$$

$$A = A_0 - \frac{b_0}{L_0} A * A$$

$$L_0 A_0 = 0$$

Analog of Yang-Feldman eq.

$$A = A_0 - \frac{b_0}{L_0}(A_0 * A_0) + \frac{b_0}{L_0}\left[\left(\frac{b_0}{L_0}(A_0 * A_0)\right) * A_0 + A_0 * \left(\frac{b_0}{L_0}(A_0 * A_0)\right)\right] + \ldots$$



A_0

Resonance

Sen

$$A_0^{new} = a e^{\pm i\varpi\cdot t}\Psi$$

$$-\omega^2 + m^2 + \mathrm{A}^{(4)} a^2 = 0$$

Problems!!!

OUTLOOK

String Field Theory

L.Bonora

Noncommutative Field Theories and
(Super) String Field Theories, **hep-th/0111208,**
I.A., **D. Belov, A.Giryavets, A.Koshelev,P.Medvedev**

SuperString Field Theory

2-nd

- ***Cubic SSFT action***
- ***Tachyon Condensation in SSFT***

3-d

- ***Rolling Tachyon***
- ***Vacuum SuperString Field Theory***

i) New BRST charge
ii) Special solutions - sliver, lump, etc.:
algebraic; surface states; Moyal representation

Super String Field Theory

(Cubic SSFT)

(Lecture II)

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OUTLOOK

String Field Theory

L.Bonora

SuperString Field Theory

2-nd
- *Cubic SSFT action*
- *Tachyon Condensation in SSFT*

1-st
- *Rolling Tachyon*

3-d
- *Vacuum SuperString Field Theory*

  i) New BRST charge
  ii) Special solutions - sliver, lump, etc.:
  algebraic; surface states; Moyal representation

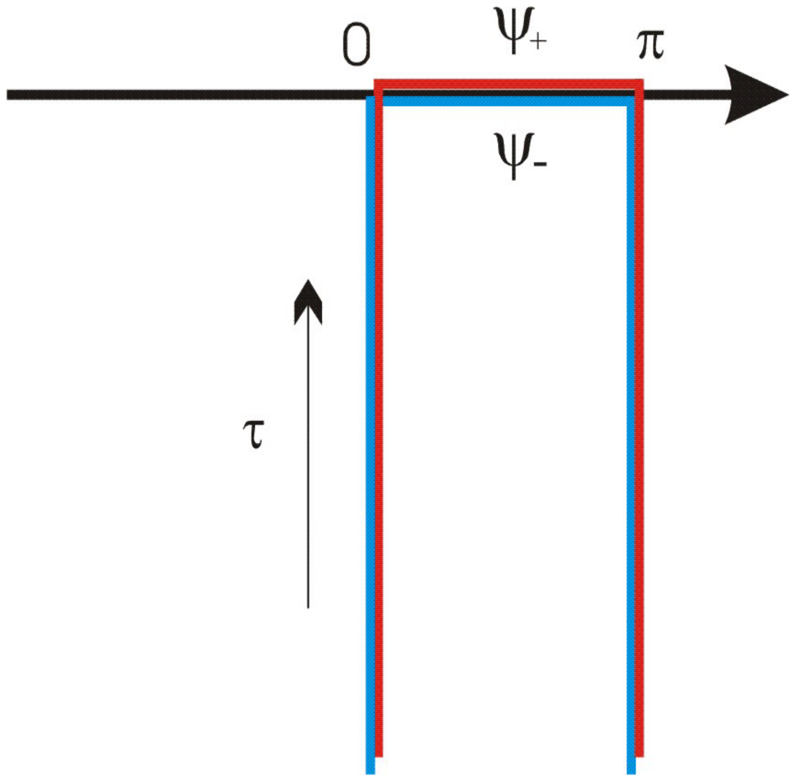
Super String Theory (NSR-formalism)

$$S = \frac{1}{2\pi\alpha'^2} \int_{-\infty}^{\infty} d\tau \int_0^{\pi} d\sigma [\frac{1}{2}\partial_\alpha X^\mu \partial^\alpha X_\mu - \frac{i}{2}\overline{\Psi}^\mu \gamma^\alpha \partial_\alpha \Psi_\mu]$$

Ψ **2dim Majorana spinor,** $\overline{\Psi} = \Psi^T \gamma^0$ $\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\gamma^1 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}$$

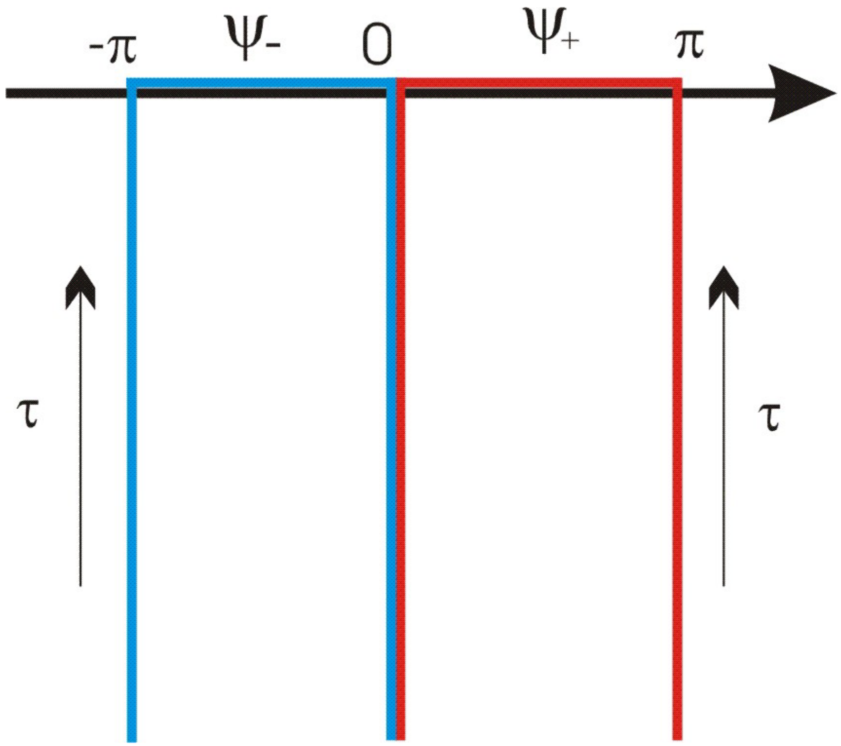
Super String Theory (NSR-formalism)



Boundary conditions:

NS: $\psi_+(\pi) = -\psi_-(\pi)$

R: $\psi_+(\pi) = \psi_-(\pi)$



$$\psi(\sigma) = \begin{cases} \psi_+(\sigma) & \sigma \in (0, \pi) \\ \psi_-(-\sigma) & \sigma \in (-\pi, 0) \end{cases}$$

Super String Theory (NSR-formalism)

$$\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} \qquad (\tau, \sigma) \Rightarrow (z, \bar{z})$$

$$S_f = -\frac{1}{2\pi\alpha'^2} \int d^2 z [\Psi_+{}^\mu \bar{\partial} \Psi_{+\mu} + \Psi_-{}^\mu \partial \Psi_{-\mu}]$$

$$\Psi_+(z) = \sum_{n \in \aleph + r} \frac{\psi_n}{z^{n+1/2}}$$

r=1/2 for R-sector;
r=0 for NS-sector

Quantization $$\{\psi_n^\mu, \psi_m^\nu\} = \eta^{\mu\nu} \delta_{n+m,0}$$

Correlators $$\langle \psi^\mu(z) \psi^\nu(w) \rangle = -\frac{\alpha'}{2} \eta^{\mu\nu} \frac{1}{z-w}$$

Identity Overlap

Gross, Jevicki

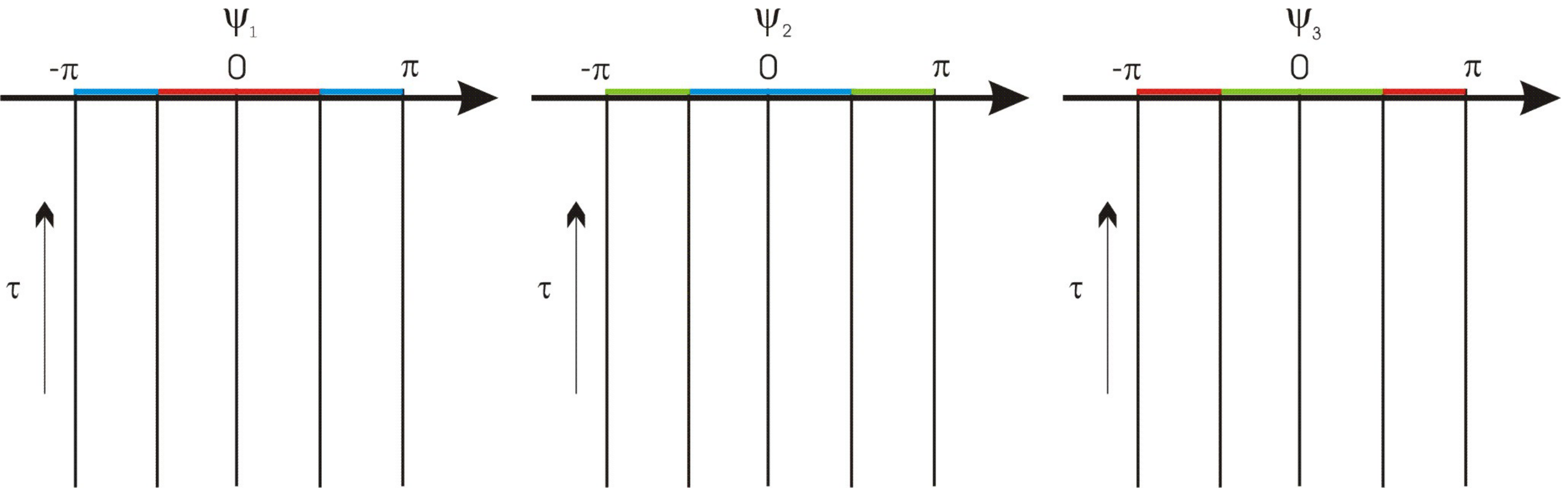
$$\psi(\sigma) = \left(\frac{\partial\sigma'}{\partial\sigma}\right)^{\frac{1}{2}} \psi(\sigma')$$

$$\psi_+(\sigma) = i\psi_+(\pi - \sigma), \quad \sigma \in \left(0, \frac{\pi}{2}\right)$$

$$\psi_-(\sigma) = -i\psi_-(\pi - \sigma), \quad \sigma \in \left(0, \frac{\pi}{2}\right)$$

$$\psi(\sigma) = \begin{cases} i\psi(\pi - \sigma) & \sigma \in \left(0, \frac{\pi}{2}\right) \\ -i\psi(-\pi - \sigma) & \sigma \in \left(-\frac{\pi}{2}, 0\right) \end{cases}$$

Three string vertex overlap



$$\psi_a(\sigma) = \begin{cases} i\psi_{a-1}(\pi - \sigma) & \sigma \in \left(0, \frac{\pi}{2}\right) \\ -i\psi_{a-1}(-\pi - \sigma) & \sigma \in \left(-\frac{\pi}{2}, 0\right) \end{cases} \qquad a = 1, 2, 3$$

Fermionic Vertices

Identity

$$|I\rangle = \exp\Big[\frac{1}{2}\sum_{r,s\in\mathbb{Z}+\frac{1}{2}} \psi_r^\dagger(-1)^{r+1/2}(CI)_{rs}\psi_s^\dagger\Big]|0\rangle$$

Three string vertex

$$|V_3\rangle_{123} = \exp\Big[\frac{1}{2}\sum_{a,b}\sum_{r,s\in\mathbb{Z}+\frac{1}{2}} \psi_r^{a\dagger}(CM^{ab})_{rs}\psi_s^{b\dagger}\Big]|0\rangle_{123}$$

Superstring (NSR-formalism); ghosts

$$S_{gh} = \frac{1}{2\pi}\int d^2 z \beta \bar{\partial} \gamma$$

$$\gamma(z) = \sum_{n \in \aleph + r} \frac{\gamma_n}{z^{n-1/2}} \qquad \beta(z) = \sum_{n \in \aleph + r} \frac{\beta_n}{z^{n+3/2}}$$

r=1/2 for R-sector;
r=0 for NS-sector

Quantization $[\gamma_m, \beta_n] = \delta_{n+m,0}$

Bosonization $\gamma = \eta e^{\phi}$ $\beta = e^{-\phi}\partial\xi$

$< e^{-2\phi} > \neq 0$

Cubic Super String Field Theory

$$S=\frac{1}{4g_0^2}[<A*QA>+\frac{2}{3}<X|A*A*A>]$$

E.Witten (1986)

Went,….

PROBLEMS **E.O.M.** $QA+XA*A=0$

Y_{-2}

I.A., Medvedev, Zubarev (1990)
Preitschopf, Thorn, Yost (1990)

$$S=\frac{1}{4g_0^2}[<Y_{-2}|A*QA>+\frac{2}{3}<Y_{-2}|A*A*A>]$$

$QA+A*A=0$ **Up to a kernal**

String Field Theory on a non-BPS brane

I.A.,Belov,Koshelev,Medvedev(2001)

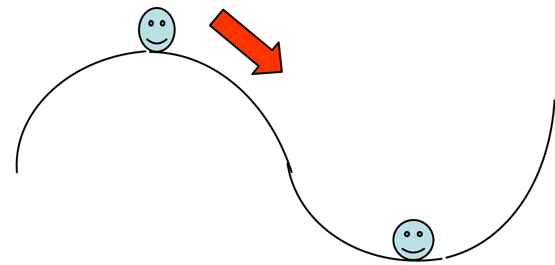
$$S=\frac{1}{4g_0^2}Tr[<\hat{Y}_{-2}|A*\hat{Q}A>+\frac{2}{3}<\hat{Y}_{-2}|A*A*A>]$$

$$A=A_+\otimes\sigma_3+A_-\otimes i\sigma_2 \qquad \hat{Q}=Q_B\otimes\sigma_3$$

	Parity	GSO
A_+	odd	+
A_-	even	-

Tachyon Condensation in SFT

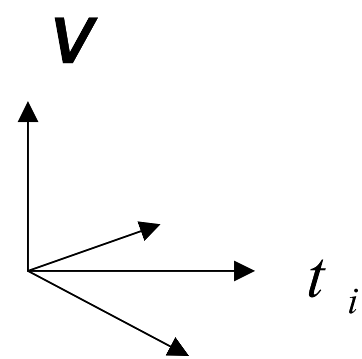
- Bosonic String - Tachyon



- Super String has no Tachyon
- Tachyon in GSO (-) sector of NS string

Kostelecky,Samuel (1989)

- Level truncation



Vertex operators in pictures –1 and 0

Level L_0+1	GSO $(-1)^F$	Name	Picture -1	Picture 0
0	+	u	-	c
1/2	-	t	$ce^{-\phi}$	$e^{\phi}\eta$
1	+	r	$c\partial c\partial\xi e^{-2\phi}$	$\partial c,\ c\partial\phi$
3/2	-	s	$c\partial\phi e^{-\phi}$	$cT_F,\partial(\eta e^{\phi}), bc\,\eta e^{\phi},\eta\partial e^{\phi}$
2	+	v_i	$\eta,\ T_F ce^{-\phi},$ $\partial\xi c\partial^2 ce^{-2\phi}$	$\partial^2 c, cT_B, cT_{\xi\eta},$ $cT_{\phi}, c\partial^2\phi, T_F e^{\phi}\eta$

$$\Phi(z) \longleftrightarrow |\Phi\rangle = \lim_{z\to 0}\Phi(z)|0\rangle$$

Berkovits,Sen,Zwiebach (2000)

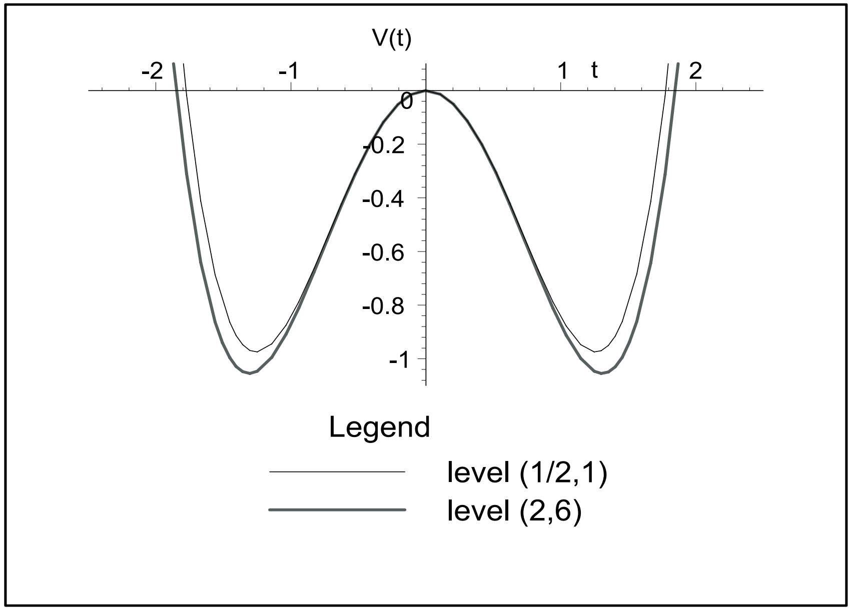
$$u,\ t,\ v_i,\ i = 1,\dots 6$$

$\Rightarrow$ *I.A.,Belov,Koshelev,Medvedev (2001)*

Tachyon Condensation in SSFT

$$V^{(1)} = \frac{1}{g_0^2 \alpha'^{p+1/2}} \left[\frac{81}{1024} t^4 - \frac{1}{4} t^2 \right]$$

$$V^{(4)} = \frac{1}{g_0^2 \alpha'^{p+1/2}} \left[\frac{5053}{69120} t^4 - \frac{1}{4} t^2 \right]$$



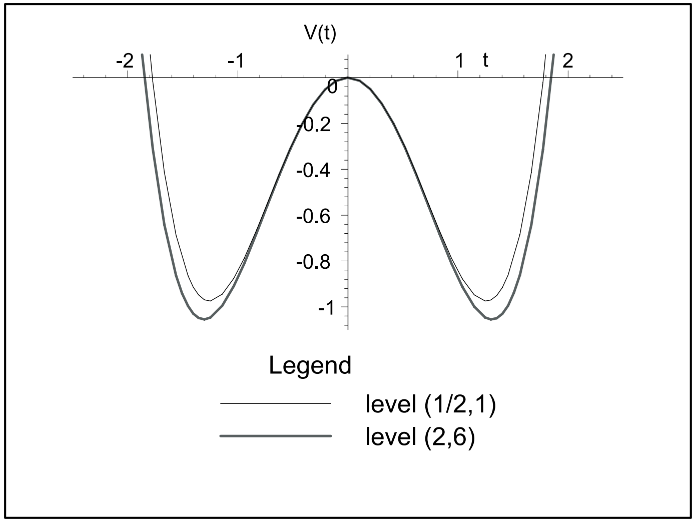
$$t_c^{(1)} = \pm 1.257$$ **97.5%**

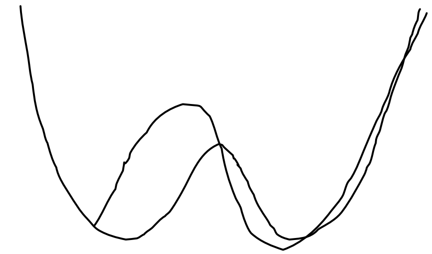
$$t_c^{(4)} = \pm 1.308$$ **105%**

For the non-polinomial Berkovic action (Berkovic, Sen, Zwiebach):

85%, **90.5%**

FAQ: cubic unbounded $\Phi^4 - \Phi^2$





$$V^{(1)} = \frac{1}{g_0^2 \alpha'^{\,p+1/2}} \left[\frac{81}{1024} t^4 - \frac{1}{4} t^2 \right]$$

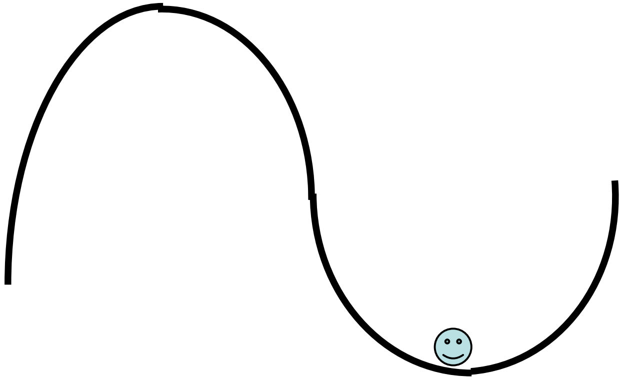
A.: Auxiliary fields

$$\frac{1}{g}\sigma^2 - 2(\varphi^2 - a^2)\sigma \qquad\qquad -g(\varphi^2 - a^2)^2$$

u, t fields

$$L^{(1)} = \frac{1}{g_0^2 \alpha'^{\,p+1/2}} [u^2 + \frac{1}{4} t^2 + \frac{1}{3\gamma^2} u t^2]$$

$$\gamma = \frac{4}{3\sqrt{3}}$$



NO OPEN STRING EXCITATIONS

VSFT

Vacuum SuperString Field Theory

(Lecture III)

I.Ya. Aref'eva
Steklov Mathematical Institute

Based on : I.A., D. Belov, A.Giryavets, A.Koshelev,
hep-th/0112214, hep-th/0201197,
hep-th/0203227, hep-th/0204239

OUTLOOK

- ***Vacuum SuperString Field Theory***

  i) New BRST charge

  ii) Special solutions - sliver, lump, etc.:
  algebraic; surface states;
  Moyal representation

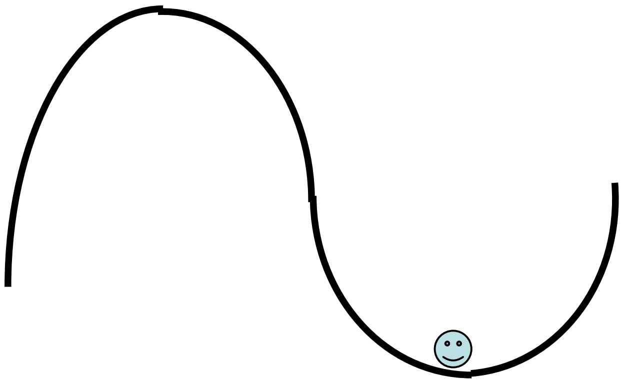
- ***Conclusion***

Sen's conjectures

- $\Delta E = T_{brane}$ **Our calculations:** **0.975** **1.058**

- **NO OPEN STRING EXCITATIONS**

- **CLOSED STRING EXCITATIONS (?)**



NO OPEN STRING EXCITATIONS

VSFT

String Field Theory on a non-BPS brane

$$S=\frac{1}{4g_0^2}Tr[<\hat{Y}_{-2}|A*\hat{Q}_B A>+\frac{2}{3}<\hat{Y}_{-2}|A*A*A>]$$

$$A=A_+\otimes\sigma_3+A_-\otimes i\sigma_2 \qquad \hat{Q}_B=Q_B\otimes\sigma_3$$

	Parity	GSO
A_+	odd	+
A_-	even	-

Vacuum String Field Theory on a non-BPS brane

I.A., Belov, Giryavets (2002)

$$S=\frac{1}{4g_0^2}Tr\,[<\hat{Y}_{-2}\,|\,A*\hat{\mathrm{Q}}\,A>+\frac{2}{3}<\hat{Y}_{-2}\,|\,A*A*A>]$$

$$A=A_+\otimes\sigma_3+A_-\otimes i\sigma_2$$

$$\hat{Q}_B=\begin{pmatrix} Q_B & 0 \\ 0 & -Q_B \end{pmatrix}$$

$$\hat{Q}=\begin{pmatrix} Q_{odd} & Q_{even} \\ -Q_{even} & -Q_{odd} \end{pmatrix}$$

Structure of new Q

$$Q = \begin{pmatrix} Q_{odd} & Q_{even} \\ -Q_{even} & -Q_{odd} \end{pmatrix}$$

SFT in the background field

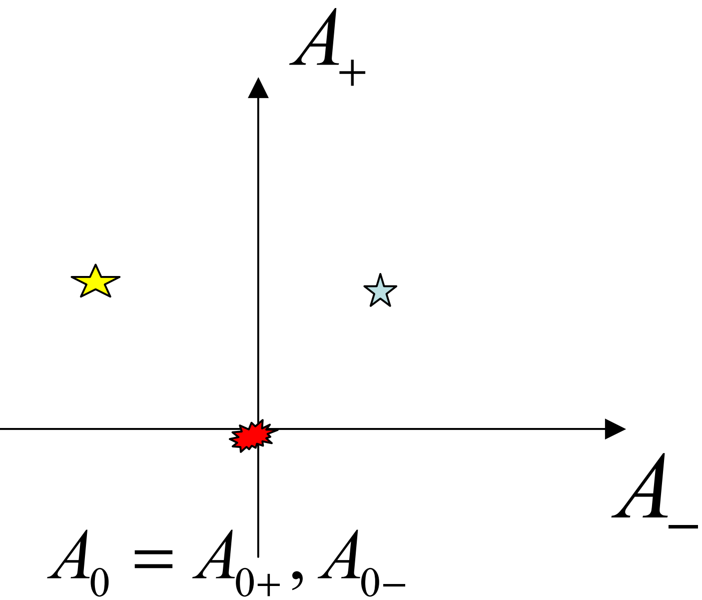
A_0 **solution to E.O.M**

$$A \Rightarrow A_0 + A$$

$$\mathbf{Q} = Q_{BRST} + \{A_0, \dots\}$$

$$\mathbf{Q} \xrightarrow{\text{field -redef.}} \mathbf{Q}_{new}$$

Ohmori



$$Q_{odd} = \frac{1}{i}\big(c(i) - c(-i)\big) + \frac{1}{2\pi i}\oint b\gamma^2(z)dz$$

$$Q_{even}^{+} = \frac{1}{i}\big(\gamma(i) - \gamma(-i)\big)$$

$$Q_{even}^{-} = \gamma(i) + \gamma(-i)$$

E.O.M.

$$QA + A * A = 0 \Rightarrow A^m + A^m * A^m = 0$$

Analog of Noncommutative Soliton in Strong Coupling Limit

Gopakumar, Minwalla, Strominger

$$\frac{1}{g}\Box\phi + \phi \star \phi - \phi = 0 \rightarrow \phi \star \phi = \phi$$

Methods of solving

$$A = A * A$$

- Algebraic method
- Surface states method
- Moyal representation
- Half-strings
- Auxiliary linear system

Algebraic Method

Identities for squeezed states

$$e^{a_i^+ S_{ij} a_j^+} \left| 0 \right\rangle$$

- **Bosonic sliver**

Rastelli, Sen, Zwiebach; Kostelecky, Potting...

- **Fermionic sliver**

$$|\Xi^\psi\rangle = \mathcal{N} \exp(-\frac{1}{2}\psi_r^\dagger(-1)^r T_{rs}\psi_s^\dagger)|0\rangle$$

$$T = \frac{1 + CI^{-1}M_{11} - \sqrt{(1 + CI^{-1}M_{11})^2 - 4M_{11}^2}}{2M_{11}}$$

Matrices M_{11} and I specify vertices

I.A., Giryavets, Medvedev; Marino, Schiappa

Conformal Sliver

Conformal map

$$f(\xi) = \arctan(\xi)$$

$$|\tilde{\Xi}^{\psi}\rangle = \tilde{\mathcal{N}} \exp(\frac{1}{2}\psi_r^\dagger \tilde{S}_{rs}\psi_s^\dagger)|0\rangle$$

$$\tilde{S}_{rs} = \oint \frac{d\xi}{2\pi\imath} \oint \frac{d\xi'}{2\pi\imath} \xi^{-r-\frac{1}{2}} \xi'^{-s-\frac{1}{2}} \times \frac{2\imath}{\sqrt{1+\xi^2}\sqrt{1+\xi'^2}} \left(\ln \left(\frac{(1+i\xi)(1-i\xi')}{(1-i\xi)(1+i\xi')} \right) \right)^{-1}$$

Comparison with algebraic sliver

L	$S_{\frac{1}{2}\frac{3}{2}}$	$S_{\frac{1}{2}\frac{7}{2}}$	$S_{\frac{1}{2}\frac{11}{2}}$	$S_{\frac{5}{2}\frac{3}{2}}$	$S_{\frac{5}{2}\frac{7}{2}}$	$S_{\frac{5}{2}\frac{11}{2}}$
30	0.13744	−0.09233	0.07098	−0.01261	0.01996	−0.02026
60	0.14667	−0.09950	0.07696	−0.01658	0.02305	−0.02286
100	0.15306	−0.10471	0.08146	−0.01940	0.02535	−0.02484
130	0.15616	−0.10729	0.08372	−0.02078	0.02650	−0.02585
Exact	0.16667	−0.11944	0.09649	−0.02500	0.03161	−0.03133

Universality of Conformal Sliver

- **Surface states** $|S^{\varphi}\rangle = U_f|0\rangle, \qquad |0\rangle -$ **conformal vacuum**
- **Conformal definition of surface states**

$$|S^{\varphi}\rangle = N\exp\left[\frac{1}{2}\varphi_{-r}S^{\varphi}_{rs}\varphi_{-s}\right]|0\rangle$$

$$\mathrm{S}^{\varphi}_{\mathrm{rs}} = \oint\frac{d\xi}{2\pi i}\oint\frac{d\xi'}{2\pi i}\,\xi^{-r-h_{\varphi}}\xi'^{-s-h_{\varphi}}\; s_{\varphi}(f(\xi), f(\xi'))$$

$$\varphi = \mathrm{X}^{\mu};\psi^{\mu};(b,c);(\beta,\gamma)$$

- **Sliver conformal map** $\mathrm{f}(\xi) = \arctan(\xi)$

- **Sliver projection equation** $|\Xi^{\varphi}\rangle = |\Xi^{\varphi}\rangle * |\Xi^{\varphi}\rangle$

Open Superstring Star in Diagonal Basis

I.A.,A.Giryavets hep-th/0204239

- **Diagonal basis**

$$\psi_{2n-1/2} = \sqrt{2}\int_0^\infty d\kappa\, v_{2n}(\kappa) e_\kappa \qquad \psi_{2n-1-1/2} = -\sqrt{2}i\int_0^\infty d\kappa\, v_{2n-1}(\kappa) o_\kappa$$

- **Three-string vertex in diagonal basis**

$$|V_3\rangle = N_V \exp\Big[\frac{1}{2}\sum_{a,b=1}^{3} \int_0^\infty d\kappa \Big(\frac{1}{2}(\mu^{ab}(\kappa) - \mu^{ba}(\kappa))(e_\kappa^{(a)+} e_\kappa^{(b)+} + o_\kappa^{(a)+} o_\kappa^{(b)+})$$
$$+ \frac{i}{2}(\mu^{ab}(\kappa) + \mu^{ba}(\kappa))(e_\kappa^{(a)+} o_\kappa^{(b)+} - o_\kappa^{(a)+} e_\kappa^{(b)+}) \Big) \Big] |0\rangle$$

- **Identity and sliver in diagonal basis**

$$|I\rangle = N_I \exp\left[i\int_0^\infty d\kappa\, j(\kappa) e_\kappa^+ o_\kappa^+ \right]|0\rangle \qquad |\Xi\rangle = N_\Xi \exp\left[i\int_0^\infty d\kappa\, T(\kappa) e_\kappa^+ o_\kappa^+ \right]|0\rangle$$

- **Spectrum of identity and sliver**

$$j(\kappa) = -\tanh\left(\frac{\pi\kappa}{4}\right) \qquad T(\kappa) = \exp\left(-\frac{\pi\kappa}{2}\right)$$

Sliver in the Moyal representation

Identity $$I(x) \sim 1$$

Sliver $$\Xi(x) \sim \exp[x\, \Theta^{-1}\, x]$$

Twisted SuperSliver

- **Superghost twisted sliver** I.A., Giryavets, Koshelev, hep-th/0203227

$$|\Xi\rangle' = U_f \; |0\rangle'_{bc} \otimes |0\rangle'_{\beta\gamma} = U_f \;\; c_1 |0\rangle_{bc} \otimes e^{-\phi(0)} |0\rangle_{\beta\gamma}$$

- **Superghost twisted sliver equation**

$$|\Xi\rangle' * |\Xi\rangle' \propto (c(i)e^{-\phi(i)} - c(-i)e^{-\phi(-i)}) |\Xi\rangle' \qquad (1)$$

- **Sliver with insertion**

$$|\widetilde{\Xi}\rangle = U_f \; |0\rangle_{bc} \otimes \gamma(0) |0\rangle_{\beta\gamma}$$

- **Picture changing**

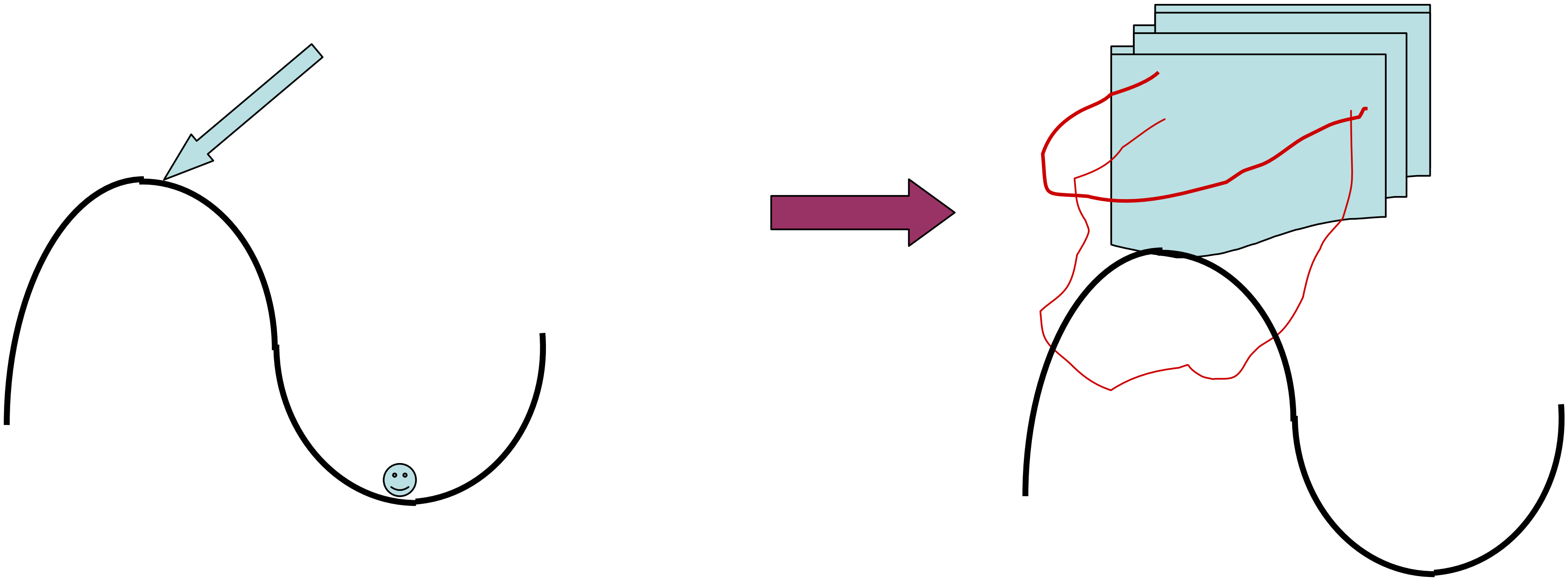
$$Q_{odd} = \tfrac{1}{i}\big(c(i) - c(-i)\big) + \tfrac{1}{2\pi i}\oint b\gamma^2(z)dz$$
$$Q^+_{even} = \tfrac{1}{i}\big(\gamma(i) - \gamma(-i)\big)$$
$$Q^-_{even} = \gamma(i) + \gamma(-i)$$

$$Y(\pm(i+\varepsilon)) |\widetilde{\Xi}\rangle = U_f \; Y(\pm i\varepsilon) \;\; |0\rangle_{bc} \otimes \gamma(0) |0\rangle_{\beta\gamma} = U_f \;\; c_1 |0\rangle_{bc} \otimes e^{-\phi(0)} |0\rangle_{\beta\gamma} = |\Xi\rangle', \;\; \varepsilon \to 0$$

$$Y(i)Y(-i) \;\; \times \;\; (1) \;\; \Rightarrow \;\; |\widetilde{\Xi}\rangle * |\widetilde{\Xi}\rangle \propto (\gamma(i) - \gamma(-i)) |\widetilde{\Xi}\rangle$$

Tests

Solution to VSFT E.O.M



Conclusion

- What we know
- What we have got
- Open problems

What we know

SSFT proposes a hard, but a surmountable way to get answers concerning non-perturbative phenomena

Two sets of basis:

i) related with spectrum of free string

ii) related with "strong coupling " regime
(may be suitable for study VSFT)

What we have got in cubic SSFT

- **Tachyon condensation**
- **Rolling tachyon near the top**
- **Vacuum SSFT** ***and some solutions***

Open Problems

★ **More tests for checking validity of VSSFT**

★★ **Other solutions (lump, kink solutions); especially with time dependence**

Use the Moyal basis to construct the tachyon condensate and other solutions

Classification of projectors in open string field algebra and its physical meaning

★★★ **Closed string excitations in VSSFT**